

# On the spectral domain approach to long-range propagation of high-frequency waves along a strip conductor above a PEC surface

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## Abstract

A generic problem of high frequency wave propagation along a metallic strip in parallel above a PEC ground plane is considered. The wave is excited by an elemental electric dipole at an arbitrary location above the PEC plane. The full wave problem, for arbitrary widths of the strip, is solved by means of a mode matching approach and expansion of the strip surface current into Chebyshev polynomials. For narrow strips, an approximate method using only longitudinal currents is derived, and compared numerically with the full wave method. Utilizing the concept of equivalent radius, the approximate method for narrow strips is evaluated numerically against results for thin circular wires. It is concluded that the approximate method is suitable for handling multiple wires in layered structures, wherefore the method has potential usefulness for estimating long range propagation of high frequency waves in wire structures like power lines and railway feeding systems, containing over-head wires and wires submerged into ground.

## 1 Introduction

The analysis of electromagnetic waves propagating along wires parallel with interfaces, separating regions having different electromagnetic properties, is of importance when considering electrical systems, like power lines [4], submerged cables [8, 11] and the power supply for railways [1]. In such power supply systems, the waves can be unintentional, triggered by internal and/or external disturbances [1, 3, 10], or intentional, for e.g. monitoring and communication purposes [2, 4, 5].

The literature on methods for analyzing parallel conductor structures is vast, wherefore we will not give a comprehensive survey here; a selection of references can be found in the monograph [25]. In a low frequency application, it is suitable to model the structure as a multiconductor transmission line (MTL), and solve the propagation problem by means of the quasi-TEM mode theory [22, 23], generalized to account for the coupling to external sources [25](chapter 1). The basic assumption is that at the highest frequency, the wavelength is large compared with the transverse dimensions of the system. For power lines and railway feeding systems one can thus reliably use MTL theory at frequencies below about 1-10 MHz. For frequencies at which the wavelength becomes comparable with the wire separations the presence of higher order radiating modes cannot be neglected. In such cases one can use full wave, or nearly full wave, methods, like generalized MTL methods [25](chapter 4) or other methods, like the Finite Difference Time-Domain method [9] and integral equation based methods [6, 7].

Typically, the full wave methods are based on numerical discretization of wires of finite length, whereby for very long wires the required numerical accuracy may yield equations that are too massive from the computational point of view. In particular, this will be the case if one wants to study electromagnetic interference (EMI) and communication at frequencies of orders from 100 MHz to several GHz, where a typical power line becomes electrically very large also in the transversal directions. Hence, in order to estimate e.g. possible distances for GHz communication along wire structures and EMI effects at the same frequencies one needs methods capable of handling very long (in terms of wavelengths) wire structures.

One example of a nearly full wave method for an infinitely long uniform MTL is presented in [14]. The MTL is located above a lossy halfspace and illuminated by a plane wave with an arbitrary direction of incidence; remotely located dipole sources are also considered. The uniformity facilitates the use of a spatial Fourier transform whereby the problem can be solved in the transform space, i.e. the spectral domain. Except for the thin wire approximation, this method is exact.

Spectral domain methods have been used frequently for studying propagation along MTL structures in the form of thin planar strip conductors on substrates, see e.g. Sect. 4.8 in [26] for a calculation of the transmission line parameters for quasi-TEM modes propagating along a microstrip line. Hence, it is of interest to investigate whether these methods can be useful also for the above mentioned power line problems. A motivation for considering methods developed for planar conductors is that if the thin wire approximation holds, and proximity effects can be neglected, a wire of quite arbitrary cross-section can be replaced with an equivalent strip conductor. For example, for a circular wire the equivalent strip has a width that is twice the diameter of the wire [21]. In [15], the two-dimensional scattering problem for a strip illuminated by a plane wave has been solved, for both polarizations of the incident electric field. Excitation/scattering problems for strip gratings using localized nearby sources have been considered in [19,20]. In the context of power/railway lines, localized sources are of relevance when studying EMI emanating from e.g. spark-overs in insulators or from pantograph arcing [27]. Another situation where localized sources are of relevance is high frequency communication, where the guided wave on the wire structure can be excited by coupling to a suitably placed antenna.

In this paper, we consider a metallic strip placed in parallel above a perfectly electrically conducting (PEC) plane. The configuration is excited by an electric dipole at an arbitrary location above the PEC plane. For the analysis, we use a spectral domain method, based on Fourier transforms in both of the horizontal directions. The analysis is systemized in terms of classical waveguide theory, where the spectral components are treated as waveguide modes in the vertical direction [28](Section 3.2b). Hence, the strip can be considered as a generalized diaphragm in a plane connecting two waveguide sections; a similar point of view has also been used in [19]. Like in [15], we will in the context of the strip model make no approximations, whereby the analysis holds for arbitrary widths of the strip, in terms of wavelengths. Furthermore, there are no restrictions on the separation distance between the wire and the PEC plane.

The paper is organized as follows. In Section 2, we formulate the problem and derive the spectral method. For clarity, the derivation is carried out in some detail, with additional details in the appendices. The result is a linear system for determining the coefficients of the spectral surface current density on the strip. In Section 3, we present numerical results. First, we consider a wide strip, whereby we have to use the exact (truly full wave) method. Then we consider a narrow strip and investigate the applicability of certain approximations for that case. Finally, we evaluate the equivalent radius concept, by comparing the results for narrow strips with results for a circular wire: analytical results for the TEM-mode approximation and full-wave results obtained by the MoM based Numerical Electromagnetics Code (NEC) [31]. The method and the results are summarized and discussed in Section 4.

## 2 Problem formulation and theory

The problem geometry is depicted in Figure 1. There is a PEC surface in the plane  $z = 0$ . In the plane  $z = a$  there is an infinitely thin PEC strip that is infinite in the  $x$ -direction and with edges at  $y = \pm h$ . At the location  $\mathbf{r}_0 = x_0\hat{\mathbf{x}} + y_0\hat{\mathbf{y}} + z_0\hat{\mathbf{z}}$  there is an electric dipole source with dipole moment  $\mathbf{p}$ . The medium in the region  $z > 0$  is homogeneous, isotropic and in general lossy, described by the complex permittivity  $\varepsilon$  and the complex permeability  $\mu$ , which are in general frequency dependent.

The problem is to find the fields and the strip surface current that results from the radiation from the dipole.



Figure 1

### 2.1 Preliminaries

We start from the Maxwell equations in a homogeneous isotropic medium:

$$\nabla \times \mathbf{E}(\mathbf{r}) = -jk\eta\mathbf{H}(\mathbf{r}), \quad (1)$$

$$\eta\nabla \times \mathbf{H}(\mathbf{r}) = +jk\mathbf{E}(\mathbf{r}) + \eta\mathbf{J}(\mathbf{r}), \quad (2)$$

where  $\mathbf{E}$  and  $\mathbf{H}$  are the electric and magnetic fields, respectively,  $k = \omega\sqrt{\varepsilon\mu}$  is the medium wave number,  $\eta = \sqrt{\mu/\varepsilon}$  is the medium wave impedance and  $\mathbf{J}$  is the source current density.  $\omega$  is the angular frequency, in a suppressed  $e^{j\omega t}$  time-dependence.

We decompose the radius vector into  $\mathbf{r} = \boldsymbol{\rho} + z\hat{\mathbf{z}}$ , where the transversal part  $\boldsymbol{\rho} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ , and introduce the following pair of Fourier-transforms:

$$\mathbf{E}(\mathbf{k}_t, z) = \int_{\mathcal{S}} \mathbf{E}(\mathbf{r}) e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}} d\mathbf{s}, \quad (3)$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi^2} \int_{\mathcal{K}} \mathbf{E}(\mathbf{k}_t, z) e^{-j\mathbf{k}_t \cdot \boldsymbol{\rho}} d^2k_t, \quad (4)$$

defined analogously for  $\mathbf{H}, \mathbf{J}$  etc.. In (3),  $d\mathbf{s} = dx dy$  and  $\mathcal{S}$  denotes the entire  $xy$ -plane. The spectral (Fourier) variable  $\mathbf{k}_t = k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}}$  is a transverse wave-vector. In (4),  $d^2k_t = dk_x dk_y$  and  $\mathcal{K}$  denotes the entire  $k_x k_y$ -plane. In the following, we refer to  $\mathbf{E}(\mathbf{r})$  as the spatial electric field and to  $\mathbf{E}(\mathbf{k}_t, z)$  as the spectral electric field, and similarly for other quantities.

## 2.2 TM- & TE-modes

In any slab region,  $z_1 < z < z_2$ , where  $\mathbf{J}(\mathbf{r}) = \mathbf{0}$ , it follows from (1) and (2) that the fields satisfy the homogeneous Helmholtz equation

$$(\nabla^2 + k^2) \begin{bmatrix} \mathbf{E}(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (5)$$

Using (4) to express  $\mathbf{E}(\mathbf{r})$  and  $\mathbf{H}(\mathbf{r})$  in (5), and using the property  $\nabla e^{-j\mathbf{k}_t \cdot \boldsymbol{\rho}} = -j\mathbf{k}_t e^{-j\mathbf{k}_t \cdot \boldsymbol{\rho}}$ , it follows that the spectral fields satisfy

$$\left( -k_t^2 + \frac{\partial^2}{\partial z^2} + k^2 \right) \begin{bmatrix} \mathbf{E}(\mathbf{k}_t, z) \\ \mathbf{H}(\mathbf{k}_t, z) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (6)$$

with solutions of the form

$$\mathbf{E}(\mathbf{k}_t, z), \mathbf{H}(\mathbf{k}_t, z) \propto e^{\mp j k_z z} \quad (7)$$

where the longitudinal wavenumber

$$k_z = \sqrt{k^2 - k_t^2} \quad (8)$$

Since any physical medium exhibits losses, it follows from the passivity conditions [30] that  $\text{Im}\{\varepsilon\} < 0$  and that  $\text{Im}\{\mu\} < 0$ , resulting in that  $k$  and  $k_z$  both are complex. The branch of the square root in (8) is always chosen such that  $\text{Im}\{k_z\} < 0$ . Hence,  $e^{-j k_z z}$  represents a solution decaying in the  $+z$ -direction, while  $e^{+j k_z z}$  represents a solution decaying in the  $-z$ -direction.

Using (4) to express the fields in Maxwell's equations (1) and (2), with  $\mathbf{J} = \mathbf{0}$  and  $z$ -dependencies of the forms  $e^{\mp j k_z z}$ , it is shown in Appendix A that the transversal spectral field components can be expressed in terms of the longitudinal components:

$$\mathbf{E}_t^\pm = \frac{-1}{k_t^2} [\pm k_z \mathbf{k}_t E_z^\pm - k \hat{\mathbf{z}} \times \mathbf{k}_t \eta H_z^\pm], \quad (9)$$

$$\eta \mathbf{H}_t^\pm = \frac{-1}{k_t^2} [\pm k_z \mathbf{k}_t \eta H_z^\pm + k \hat{\mathbf{z}} \times \mathbf{k}_t E_z^\pm], \quad (10)$$

where  $\pm$  refers to the  $\pm z$ -decaying solutions.

From the structures of (9) and (10), we find it convenient to introduce the dimensionless functions

$$\mathbf{f}(\mathbf{k}_t) = \frac{k_z}{k_t^2} \mathbf{k}_t, \quad (11)$$

$$\mathbf{f}^\pm(\mathbf{k}_t) = \mathbf{f}(\mathbf{k}_t) \mp \hat{\mathbf{z}}, \quad (12)$$

$$\mathbf{g}(\mathbf{k}_t) = \frac{k}{k_t^2} \hat{\mathbf{z}} \times \mathbf{k}_t, \quad (13)$$

subject to

$$\mathbf{f}^\pm(\mathbf{k}_t) \cdot \mathbf{g}(\mathbf{k}_t) = \mathbf{f}(\mathbf{k}_t) \cdot \mathbf{g}(\mathbf{k}_t) = 0. \quad (14)$$

Hence, the transverse magnetic (TM) parts of the spectral fields are written

$$\mathbf{E}^{\text{TM}}(\mathbf{k}_t, z) = A^+(\mathbf{k}_t) \mathbf{f}^+(\mathbf{k}_t) e^{-jk_z z} + A^-(\mathbf{k}_t) \mathbf{f}^-(\mathbf{k}_t) e^{+jk_z z} \quad (15)$$

$$\eta \mathbf{H}^{\text{TM}}(\mathbf{k}_t, z) = \mathbf{g}(\mathbf{k}_t) [A^+(\mathbf{k}_t) e^{-jk_z z} - A^-(\mathbf{k}_t) e^{+jk_z z}] \quad (16)$$

while the transverse electric (TE) parts of the spectral fields are written

$$\mathbf{E}^{\text{TE}}(\mathbf{k}_t, z) = \mathbf{g}(\mathbf{k}_t) [B^+(\mathbf{k}_t) e^{-jk_z z} + B^-(\mathbf{k}_t) e^{+jk_z z}] \quad (17)$$

$$\eta \mathbf{H}^{\text{TE}}(\mathbf{k}_t, z) = -B^+(\mathbf{k}_t) \mathbf{f}^+(\mathbf{k}_t) e^{-jk_z z} + B^-(\mathbf{k}_t) \mathbf{f}^-(\mathbf{k}_t) e^{+jk_z z} \quad (18)$$

For each value of  $\mathbf{k}_t$  there is one TM-mode and one TE-mode<sup>1</sup>.  $A^\pm(\mathbf{k}_t)$  and  $B^\pm(\mathbf{k}_t)$  are mode coefficients, which are determined by the sources and the boundary conditions. Once the mode coefficients have been determined, the spatial fields are determined using (4), resulting in

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi^2} \int_{\mathcal{K}} [\mathbf{E}^{\text{TM}}(\mathbf{k}_t, z) + \mathbf{E}^{\text{TE}}(\mathbf{k}_t, z)] e^{-j\mathbf{k}_t \cdot \boldsymbol{\rho}} d^2 k_t, \quad (19)$$

$$\mathbf{H}(\mathbf{r}) = \frac{1}{4\pi^2} \int_{\mathcal{K}} [\mathbf{H}^{\text{TM}}(\mathbf{k}_t, z) + \mathbf{H}^{\text{TE}}(\mathbf{k}_t, z)] e^{-j\mathbf{k}_t \cdot \boldsymbol{\rho}} d^2 k_t. \quad (20)$$

## 2.3 The spectral fields from the dipole, the ground plane and the strip

In Appendix B we show that the spectral fields from the dipole become

$$\mathbf{E}^{(d)}(\mathbf{k}_t, z) = A_1(\mathbf{k}_t, z) [\mathbf{H}(z - z_0) \mathbf{f}^+(\mathbf{k}_t) + \mathbf{H}(z_0 - z) \mathbf{f}^-(\mathbf{k}_t)] + B_1(\mathbf{k}_t, z) \mathbf{g}(\mathbf{k}_t) \quad (21)$$

$$\eta \mathbf{H}^{(d)}(\mathbf{k}_t, z) = \text{sgn}(z - z_0) \cdot [A_1(\mathbf{k}_t, z) \mathbf{g}(\mathbf{k}_t) - B_1(\mathbf{k}_t, z) [\mathbf{H}(z - z_0) \mathbf{f}^+(\mathbf{k}_t) + \mathbf{H}(z_0 - z) \mathbf{f}^-(\mathbf{k}_t)]] \quad (22)$$

where  $\mathbf{H}()$  denotes the Heaviside step-function,  $\text{sgn}()$  denotes the signum-function, and where for the principal directions of the dipole

$$\mathbf{p} = p\hat{\mathbf{x}} \Rightarrow \begin{cases} A_1(\mathbf{k}_t, z) = -\frac{jp k_x}{2\varepsilon} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}_0} e^{-jk_z |z - z_0|} \\ B_1(\mathbf{k}_t, z) = \frac{jp k_y}{2\varepsilon k_z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}_0} e^{-jk_z |z - z_0|} \end{cases} \quad (23)$$

$$\mathbf{p} = p\hat{\mathbf{y}} \Rightarrow \begin{cases} A_1(\mathbf{k}_t, z) = -\frac{jp k_y}{2\varepsilon} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}_0} e^{-jk_z |z - z_0|} \\ B_1(\mathbf{k}_t, z) = -\frac{jp k_x}{2\varepsilon k_z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}_0} e^{-jk_z |z - z_0|} \end{cases} \quad (24)$$

$$\mathbf{p} = p\hat{\mathbf{z}} \Rightarrow \begin{cases} A_1(\mathbf{k}_t, z) = \text{sgn}(z - z_0) \frac{jp k_t^2}{2\varepsilon k_z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}_0} e^{-jk_z |z - z_0|} \\ B_1(\mathbf{k}_t, z) = 0 \end{cases} \quad (25)$$

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<sup>1</sup>When  $\mathbf{k}_t = \mathbf{0}$  there are no longitudinal fields, and instead two degenerate TEM-modes (with  $k_z = k$ ).

When enforcing the boundary conditions in a multilayered structure, it suffices to consider the transversal fields. For the dipole, the transversal fields become

$$\mathbf{E}_t^{(d)}(\mathbf{k}_t, z) = A_1(\mathbf{k}_t, z) \mathbf{f}(\mathbf{k}_t) + B_1(\mathbf{k}_t, z) \mathbf{g}(\mathbf{k}_t) \quad (26)$$

$$\eta \mathbf{H}_t^{(d)}(\mathbf{k}_t, z) = \text{sgn}(z - z_0) [A_1(\mathbf{k}_t, z) \mathbf{g}(\mathbf{k}_t) - B_1(\mathbf{k}_t, z) \mathbf{f}(\mathbf{k}_t)] \quad (27)$$

The transversal fields originating from the charges and currents on the ground plane at  $z = 0$  decay away from the ground plane and are for  $z > 0$  thus written

$$\mathbf{E}_t^{(0)}(\mathbf{k}_t, z) = [A_0(\mathbf{k}_t) \mathbf{f}(\mathbf{k}_t) + B_0(\mathbf{k}_t) \mathbf{g}(\mathbf{k}_t)] e^{-jk_z z} \quad (28)$$

$$\eta \mathbf{H}_t^{(0)}(\mathbf{k}_t, z) = [A_0(\mathbf{k}_t) \mathbf{g}(\mathbf{k}_t) - B_0(\mathbf{k}_t) \mathbf{f}(\mathbf{k}_t)] e^{-jk_z z} \quad (29)$$

The spectral transversal fields originating from the charges and currents on the strip in the plane  $z = a$  are written

$$\mathbf{E}_t^{(s)}(\mathbf{k}_t, z) = [A_2(\mathbf{k}_t) \mathbf{f}(\mathbf{k}_t) + B_2(\mathbf{k}_t) \mathbf{g}(\mathbf{k}_t)] e^{-jk_z |z-a|} \quad (30)$$

$$\eta \mathbf{H}_t^{(s)}(\mathbf{k}_t, z) = \text{sgn}(z - a) [A_2(\mathbf{k}_t) \mathbf{g}(\mathbf{k}_t) - B_2(\mathbf{k}_t) \mathbf{f}(\mathbf{k}_t)] e^{-jk_z |z-a|} \quad (31)$$

Note that since  $\mathbf{f}(\mathbf{k}_t) \cdot \mathbf{g}(\mathbf{k}_t) = 0$ , (30) is the unique Ansatz to enforce the continuity of  $\mathbf{E}_t^{(s)}(\mathbf{k}_t, z)$  across the plane  $z = a$ .

## 2.4 Boundary conditions

The free surface current density  $\mathbf{K}(\boldsymbol{\rho})$  on the strip has the corresponding spectral surface current density

$$\mathbf{K}(\mathbf{k}_t) = \int_S \mathbf{K}(\boldsymbol{\rho}) e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}} d\mathbf{s} \quad (32)$$

Applying the boundary condition  $\mathbf{H}_t^{(s)}(\mathbf{k}_t, z = a^+) - \mathbf{H}_t^{(s)}(\mathbf{k}_t, z = a^-) = \mathbf{K}(\mathbf{k}_t) \times \hat{\mathbf{z}}$ , we obtain from (31) that

$$2A_2 \mathbf{g} - 2B_2 \mathbf{f} = \eta \mathbf{K} \times \hat{\mathbf{z}} \quad (33)$$

Utilizing (11), (13) and (14), (33) yields

$$A_2 = \frac{\mathbf{g} \cdot (\eta \mathbf{K} \times \hat{\mathbf{z}})}{2g^2} = \frac{(\hat{\mathbf{z}} \times \mathbf{g}) \cdot (\eta \mathbf{K})}{2g^2} = -\frac{\mathbf{k}_t \cdot (\eta \mathbf{K})}{2k} = -\eta \frac{k_x K_x + k_y K_y}{2k} \quad (34)$$

$$B_2 = -\frac{\mathbf{f} \cdot (\eta \mathbf{K} \times \hat{\mathbf{z}})}{2f^2} = -\frac{(\hat{\mathbf{z}} \times \mathbf{f}) \cdot (\eta \mathbf{K})}{2f^2} = -\frac{(\hat{\mathbf{z}} \times \mathbf{k}_t) \cdot (\eta \mathbf{K})}{2k_z} = -\eta \frac{k_x K_y - k_y K_x}{2k_z} \quad (35)$$

The condition  $\mathbf{E}_t = \mathbf{E}_t^{(d)} + \mathbf{E}_t^{(0)} + \mathbf{E}_t^{(s)} = \mathbf{0}$  at the plane  $z = 0$ , together with (14), (26), (28) and (30), yields

$$A_0 = -A_1(z=0) - A_2 e^{-jk_z a} \quad (36)$$

$$B_0 = -B_1(z=0) - B_2 e^{-jk_z a} \quad (37)$$

Thus, using (26), (28), (30), (34)-(37), (11) and (13), we obtain that in the plane  $z = a$  the transversal spectral electric field becomes

$$\begin{aligned}
\mathbf{E}_{ta}(\mathbf{k}_t) &= [A_1(\mathbf{k}_t, a) - A_1(\mathbf{k}_t, 0) e^{-jk_z a} + A_2(\mathbf{k}_t) (1 - e^{-j2k_z a})] \mathbf{f}(\mathbf{k}_t) \\
&+ [B_1(\mathbf{k}_t, a) - B_1(\mathbf{k}_t, 0) e^{-jk_z a} + B_2(\mathbf{k}_t) (1 - e^{-j2k_z a})] \mathbf{g}(\mathbf{k}_t) \\
&= \left\{ \eta (1 - e^{-j2k_z a}) \frac{k_x k_y K_y(\mathbf{k}_t) - (k^2 - k_x^2) K_x(\mathbf{k}_t)}{2k k_z} \right. \\
&\quad \left. + \frac{k_x k_z (A_1(\mathbf{k}_t, a) - A_1(\mathbf{k}_t, 0) e^{-jk_z a}) + k k_y (-B_1(\mathbf{k}_t, a) + B_1(\mathbf{k}_t, 0) e^{-jk_z a})}{k_x^2 + k_y^2} \right\} \hat{\mathbf{x}} \\
&+ \left\{ \eta (1 - e^{-j2k_z a}) \frac{k_x k_y K_x(\mathbf{k}_t) - (k^2 - k_y^2) K_y(\mathbf{k}_t)}{2k k_z} \right. \\
&\quad \left. + e^{-jk_z a} \frac{k_y k_z (A_1(\mathbf{k}_t, a) - A_1(\mathbf{k}_t, 0) e^{-jk_z a}) + k k_x (B_1(\mathbf{k}_t, a) - B_1(\mathbf{k}_t, 0) e^{-jk_z a})}{k_x^2 + k_y^2} \right\} \hat{\mathbf{y}}
\end{aligned} \tag{38}$$

## 2.5 Spectral surface current densities

In the plane  $z = a$ , the spatial surface current density  $\mathbf{K}(x, y)$  is confined to the strip-conductor, with edges at  $y = \pm h$ . Explicitly taking into account the singular behaviour at the edges (see e.g. formula (4.13) in [33]), the components of the surface current density are expanded as

$$K_x(x, y) = \frac{H(h - |y|)}{\sqrt{1 - y^2/h^2}} \sum_{n=0}^{\infty} c_n(x) T_n\left(\frac{y}{h}\right) \tag{39}$$

$$\begin{aligned}
K_y(x, y) &= H(h - |y|) \sqrt{1 - y^2/h^2} \sum_{n=0}^{\infty} d_n(x) U_n\left(\frac{y}{h}\right) \\
&= \frac{H(h - |y|)}{2\sqrt{1 - y^2/h^2}} \sum_{n=0}^{\infty} d_n(x) \left[ T_n\left(\frac{y}{h}\right) - T_{n+2}\left(\frac{y}{h}\right) \right]
\end{aligned} \tag{40}$$

where  $T_n()$  and  $U_n()$  denote the  $n$ :th order Chebyshev polynomials of first and second kind, respectively.  $\{c_n(x), d_n(x)\}_{n=0}^{\infty}$  are coefficient functions to be determined.

Inserting (39) and (40) into (32) and using formula 7.355 in [24], the components of the



spectral surface current density become

$$\begin{aligned}
K_x(\mathbf{k}_t) &= \sum_{n=0}^{\infty} c_n(k_x) \int_{-h}^h \frac{T_n(y/h)}{\sqrt{1-y^2/h^2}} e^{jk_y y} dy \\
&= \pi h \sum_{n=0}^{\infty} (-1)^n [c_{2n}(k_x) J_{2n}(k_y h) + j c_{2n+1}(k_x) J_{2n+1}(k_y h)]
\end{aligned} \tag{41}$$

$$\begin{aligned}
K_y(\mathbf{k}_t) &= \frac{1}{2} \sum_{n=0}^{\infty} d_n(k_x) \int_{-h}^h \frac{T_n(y/h) - T_{n+2}(y/h)}{\sqrt{1-y^2/h^2}} e^{jk_y y} dy = \\
&= \frac{\pi h}{2} \sum_{n=0}^{\infty} (-1)^n \left[ d_{2n}(k_x) \{J_{2n}(k_y h) + J_{2n+2}(k_y h)\} \right. \\
&\quad \left. + j d_{2n+1}(k_x) \{J_{2n+1}(k_y h) + J_{2n+3}(k_y h)\} \right] \\
&= \pi h \sum_{n=0}^{\infty} (-1)^n \left[ d_{2n}(k_x) \frac{(2n+1) J_{2n+1}(k_y h)}{k_y h} + j d_{2n+1}(k_x) \frac{(2n+2) J_{2n+2}(k_y h)}{k_y h} \right]
\end{aligned} \tag{42}$$

where the spectral coefficients

$$c_n(k_x) = \int_{-\infty}^{\infty} c_n(x) e^{jk_x x} dx, \quad d_n(k_x) = \int_{-\infty}^{\infty} d_n(x) e^{jk_x x} dx \tag{43}$$

and  $J_n()$  denotes the  $n$ :th order Bessel function.

## 2.6 Equation system for the spectral coefficients

In the plane  $z = a$ , the spatial transversal electric field becomes

$$\mathbf{E}_{ta}(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} e^{-jk'_x x} dk'_x \int_{-\infty}^{\infty} e^{-jk_y y} dk_y \mathbf{E}_{ta}(k'_x, k_y) \tag{44}$$

On the strip,  $(-\infty < x < \infty) \cap (-h < y < h) \cap (z = a)$ , we have  $\mathbf{E}_{ta}(x, y) = \mathbf{0}$ . According to equation (4.9) in [33], it holds in the vicinity of an edge that  $E_x \propto \sqrt{r}$ ,  $E_y \propto 1/\sqrt{r}$ , where  $r$  is the distance to the edge. Taking into account these properties, we enforce the vanishing tangential field on the strip through the following testing procedures:

$$\int_{-\infty}^{\infty} e^{jk_x x} dx \int_{-h}^h \frac{T_m(y/h)}{\sqrt{1-y^2/h^2}} E_x(x, y) dy = 0, \quad -\infty < k_x < \infty, \quad m = 0, 1, \dots \tag{45}$$

$$\int_{-\infty}^{\infty} e^{jk_x x} dx \int_{-h}^h \sqrt{1-y^2/h^2} U_m(y/h) E_y(x, y) dy = 0, \quad -\infty < k_x < \infty, \quad m = 0, 1, \dots \tag{46}$$

When inserting (44) into (45) and (46), the spatial integrals result in

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j(k_x - k'_x)x} dx = \delta(k_x - k'_x) \quad (47)$$

$$\int_{-h}^h \frac{T_m(y/h)}{\sqrt{1 - y^2/h^2}} e^{-jk_y y} dy \propto J_m(k_y h) \quad (48)$$

$$\int_{-h}^h \sqrt{1 - y^2/h^2} U_m(y/h) e^{-jk_y y} dy \propto (m+1) \frac{J_{m+1}(k_y h)}{k_y h} \quad (49)$$

Hence, (45) and (46) imply the following results in the spectral domain:

$$\int_{-\infty}^{\infty} E_x(k_x, k_y) J_m(k_y h) dk_y = 0, \quad m = 0, 1, \dots \quad (50)$$

$$\int_{-\infty}^{\infty} E_y(k_x, k_y) \frac{J_{m+1}(k_y h)}{k_y h} dk_y = 0, \quad m = 0, 1, \dots \quad (51)$$

which, using (38) and (23)-(25), become

$$\begin{aligned} & \int_{-\infty}^{\infty} (1 - e^{-j2k_z a}) \frac{(k^2 - k_x^2) K_x(\mathbf{k}_t) - k_x k_y K_y(\mathbf{k}_t)}{2k k_z} J_m(k_y h) dk_y \\ &= \frac{j p}{2\varepsilon \eta} e^{jk_x x_0} \int_{-\infty}^{\infty} e^{jk_y y_0} J_m(k_y h) dk_y \left\{ \begin{array}{ll} -\frac{k^2 - k_x^2}{k_z} [e^{-jk_z |a - z_0|} - e^{-jk_z (a + z_0)}] & : \mathbf{p} = p\hat{\mathbf{x}} \\ \frac{k_x k_y}{k_z} [e^{-jk_z |a - z_0|} - e^{-jk_z (a + z_0)}] & : \mathbf{p} = p\hat{\mathbf{y}} \\ k_x [\text{sgn}(a - z_0) e^{-jk_z |a - z_0|} + e^{-jk_z (a + z_0)}] & : \mathbf{p} = p\hat{\mathbf{z}} \end{array} \right. \quad (52) \end{aligned}$$

$$\begin{aligned} & \int_{-\infty}^{\infty} (1 - e^{-j2k_z a}) \frac{(k^2 - k_y^2) K_y(\mathbf{k}_t) - k_x k_y K_x(\mathbf{k}_t)}{2k k_z} \frac{J_{m+1}(k_y h)}{k_y h} dk_y \\ &= \frac{j p}{2\varepsilon \eta} e^{jk_x x_0} \int_{-\infty}^{\infty} e^{jk_y y_0} J_{m+1}(k_y h) dk_y \left\{ \begin{array}{ll} \frac{k_x}{k_z h} [e^{-jk_z |a - z_0|} - e^{-jk_z (a + z_0)}] & : \mathbf{p} = p\hat{\mathbf{x}} \\ -\frac{k^2 - k_y^2}{k_y k_z h} [e^{-jk_z |a - z_0|} - e^{-jk_z (a + z_0)}] & : \mathbf{p} = p\hat{\mathbf{y}} \\ \frac{1}{h} [\text{sgn} |a - z_0| e^{-jk_z |a - z_0|} + e^{-jk_z (a + z_0)}] & : \mathbf{p} = p\hat{\mathbf{z}} \end{array} \right. \quad (53) \end{aligned}$$

Next, we insert (41) and (42) into (52) and (53), and utilize the even and odd parities of

integer order Bessel functions, to obtain the following systems for  $\{c_{2n}(k_x), d_{2n+1}(k_x)\}_{n=0}^{\infty}$ :

$$\begin{aligned} & \pi h \sum_{n=0}^{\infty} (-1)^n \int_0^{\infty} \frac{1 - e^{-j2k_z a}}{k k_z} \left\{ (k^2 - k_x^2) J_{2m}(k_y h) J_{2n}(k_y h) c_{2n}(k_x) \right. \\ & \quad \left. - j \frac{k_x}{h} (2n+2) J_{2m}(k_y h) J_{2n+2}(k_y h) d_{2n+1}(k_x) \right\} dk_y \\ &= \frac{j p}{\varepsilon \eta} e^{j k_x x_0} \int_0^{\infty} J_{2m}(k_y h) dk_y \begin{cases} -\frac{k^2 - k_x^2}{k_z} [e^{-j k_z |a-z_0|} - e^{-j k_z (a+z_0)}] \cos(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{x}} \\ \frac{k_x k_y}{k_z} [e^{-j k_z |a-z_0|} - e^{-j k_z (a+z_0)}] j \sin(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{y}} \\ k_x [\operatorname{sgn}(a - z_0) e^{-j k_z |a-z_0|} + e^{-j k_z (a+z_0)}] \cos(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{z}} \end{cases} \quad (54) \end{aligned}$$

$$\begin{aligned} & \pi h \sum_{n=0}^{\infty} (-1)^n \int_0^{\infty} \frac{1 - e^{-j2k_z a}}{k k_z} \left\{ j \frac{k^2 - k_y^2}{k_y^2 h^2} (2n+2) J_{2m+2}(k_y h) J_{2n+2}(k_y h) d_{2n+1}(k_x) \right. \\ & \quad \left. - \frac{k_x}{h} J_{2m+2}(k_y h) J_{2n}(k_y h) c_{2n}(k_x) \right\} dk_y \\ &= \frac{j p}{\varepsilon \eta} e^{j k_x x_0} \int_0^{\infty} J_{2m+2}(k_y h) dk_y \begin{cases} \frac{k_x}{k_z h} [e^{-j k_z |a-z_0|} - e^{-j k_z (a+z_0)}] \cos(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{x}} \\ -\frac{k^2 - k_y^2}{k_y k_z h} [e^{-j k_z |a-z_0|} - e^{-j k_z (a+z_0)}] j \sin(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{y}} \\ \frac{1}{h} [\operatorname{sgn}(a - z_0) e^{-j k_z |a-z_0|} + e^{-j k_z (a+z_0)}] \cos(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{z}} \end{cases} \quad (55) \end{aligned}$$

and into the following systems for  $\{c_{2n+1}(k_x), d_{2n}(k_x)\}_{n=0}^{\infty}$ :

$$\begin{aligned} & \pi h \sum_{n=0}^{\infty} (-1)^n \int_0^{\infty} \frac{1 - e^{-j2k_z a}}{k k_z} \left\{ j (k^2 - k_x^2) J_{2m+1}(k_y h) J_{2n+1}(k_y h) c_{2n+1}(k_x) \right. \\ & \quad \left. - \frac{k_x}{h} (2n+1) J_{2m+1}(k_y h) J_{2n+1}(k_y h) d_{2n}(k_x) \right\} dk_y \\ &= \frac{j p}{\varepsilon \eta} e^{j k_x x_0} \int_0^{\infty} J_{2m+1}(k_y h) dk_y \begin{cases} -\frac{k^2 - k_x^2}{k_z} [e^{-j k_z |a-z_0|} - e^{-j k_z (a+z_0)}] j \sin(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{x}} \\ \frac{k_x k_y}{k_z} [e^{-j k_z |a-z_0|} - e^{-j k_z (a+z_0)}] \cos(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{y}} \\ k_x [\operatorname{sgn}(a - z_0) e^{-j k_z |a-z_0|} + e^{-j k_z (a+z_0)}] j \sin(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{z}} \end{cases} \quad (56) \end{aligned}$$

$$\begin{aligned}
& \pi h \sum_{n=0}^{\infty} (-1)^n \int_0^{\infty} \frac{1 - e^{-j2k_z a}}{k k_z} \left\{ \frac{k^2 - k_y^2}{k_y^2 h^2} (2n+1) J_{2m+1}(k_y h) J_{2n+1}(k_y h) d_{2n}(k_x) \right. \\
& \quad \left. - j \frac{k_x}{h} J_{2m+1}(k_y h) J_{2n+1}(k_y h) c_{2n+1}(k_x) \right\} dk_y \\
& = \frac{j p}{\varepsilon \eta} e^{j k_x x_0} \int_0^{\infty} J_{2m+1}(k_y h) dk_y \left\{ \begin{array}{ll} \frac{k_x}{k_z h} [e^{-j k_z |a-z_0|} - e^{-j k_z (a+z_0)}] j \sin(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{x}} \\ - \frac{k^2 - k_y^2}{k_y k_z h} [e^{-j k_z |a-z_0|} - e^{-j k_z (a+z_0)}] \cos(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{y}} \\ \frac{1}{h} [\text{sgn}(a - z_0) e^{-j k_z |a-z_0|} + e^{-j k_z (a+z_0)}] j \sin(k_y y_0) & : \mathbf{p} = p \hat{\mathbf{z}} \end{array} \right. \\
& \hspace{15em} (57)
\end{aligned}$$

Summarizing, (54)-(57) are infinitely large linear equation systems for the spectral coefficients  $c_n(k_x)$ ,  $d_n(k_x)$ . The elements in the system matrices and driving terms are obtained through various integrals with respect to the spectral variable  $k_y$ , with the other spectral variable  $k_x$  as a parameter.

For the numerical solution, (54)-(57) are truncated into a finite number of coefficients and testing functions, and solved for different values of  $k_x$ . With the spectral coefficients determined, the spectral surface current density follows from (41) and (42), the mode coefficients from (34)-(37), and the spectral fields from (15)-(18). The details of the numerical evaluation of the spectral integrals are described in Appendix C. Finally, the spatial fields follow from (19) and (20).

The coefficients for the spatial surface current density follow from the inverse Fourier transform of (43):

$$c_n(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} c_n(k_x) e^{-j k_x x} dk_x, \quad d_n(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d_n(k_x) e^{-j k_x x} dk_x \quad (58)$$

With  $T_0 = 1$ , it follows from (39) and the orthogonality relation for the Chebyshev polynomials that the total current flowing along the strip becomes

$$\begin{aligned}
I(x) &= \int_{-\infty}^{\infty} K_x(x, y) dy = \int_{-h}^h \frac{dy}{\sqrt{1 - y^2/h^2}} \sum_{n=0}^{\infty} c_n(x) T_n\left(\frac{y}{h}\right) \\
&= h \sum_{n=0}^{\infty} c_n(x) \int_{-1}^1 \frac{T_0(u) T_n(u)}{\sqrt{1 - u^2}} du = h\pi \sum_{n=0}^{\infty} c_n(x) \delta_{n0} = \pi h c_0(x) \quad (59)
\end{aligned}$$

### 3 Numerical results

In this section, we evaluate the method through several numerical examples. In all examples, the dipole source has the  $x$ -coordinate  $x_0 = 0$  m, the frequency  $f = 300$  MHz, and the medium parameters  $\varepsilon = \varepsilon_0 (1 - j10^{-5})$ ,  $\mu = \mu_0 (1 - j10^{-5})$ . Hence, it follows that the wavelength  $\lambda \approx 1$  m and that attenuation due to medium losses have minor impact within distances of the order of some hundred meters<sup>2</sup>.

<sup>2</sup>In more realistic cases, attenuation is due to a finite ground conductivity that will mask the losses in the air. The simple PEC-model serves to not obscure the main results with too massive algebra.

### 3.1 Surface currents on a wide strip

Here, we consider a comparatively wide strip with the half-width  $h = 0.5$  m. The strip is at the height  $a = 1$  m and the dipole source is at the height  $z_0 = 0.5$  m, above the PEC plane. The required number of spectral coefficients scales with  $h/\lambda$  and here we use  $m_{\max} = 3$  in (54)-(57), i.e. the sought coefficients are  $\{c_n, d_n\}_{n=0}^7$ .

For graphical clarity and space reasons, we only consider the portion  $-3 \text{ m} < x < 3 \text{ m}$  of the strip, and the results shown are only for the real part (one particular instantaneous value) of the surface current density.

In cases with the dipole moment in the  $x$ -direction, the field will be purely TM in the  $x$ -direction, whereby  $K_y = 0$ . This extra numerical check resulted in  $K_y$ -values that were about five orders in magnitude smaller than the  $K_x$ -values. Hence, for  $x$ -directed dipoles only results for  $K_x$  are presented.

We check the symmetries expected when the dipole is centered with the strip at  $y_0 = 0$ . The results are shown in Figure 2. We see that  $K_x$  and  $K_y$  exhibit the symmetries expected, due to the locations and orientations of the dipole sources. In Figure 2(b), we see that the  $y$ -directed dipole yields odd symmetry in  $K_x$ , resulting in a zero net current flowing along the strip (cf. (59)). Also,  $K_y$  dominates over  $K_x$ , since at the strip the exciting dipole electric field is predominantly in the  $y$ -direction. For the  $z$ -directed dipole, we see in Figure 2(c) that, except for in the region above the dipole,  $K_x$  dominates over  $K_y$ .

### 3.2 Surface currents on a narrow strip

Here, we consider a comparatively narrow strip with the half-width  $h = 0.1$  m. The strip is at the height  $a = 6$  m and the dipole source is at the height  $z_0 = 5.5$  m, above the PEC plane, and off-centered at  $y_0 = 0.5$  m. Here we use  $m_{\max} = 1$  in (54)-(57), i.e. the sought coefficients are  $\{c_n, d_n\}_{n=0}^3$ .

The conjecture is that for a narrow strip the longitudinal<sup>3</sup> current dominates over the transverse current, i.e.  $|K_x| \gg |K_y|$ . Since an  $x$ -directed dipole yields  $K_y = 0$ , results for that case are omitted, due to space reasons. The results for  $y$ - and  $z$ -directed dipoles, shown in 3 Figures 3(a) and 3(b) respectively, show that for both the  $y$ -directed and the  $z$ -directed dipole,  $K_x$  dominates over  $K_y$ , which supports the conjecture that on a narrow strip the surface current density is predominantly longitudinal.

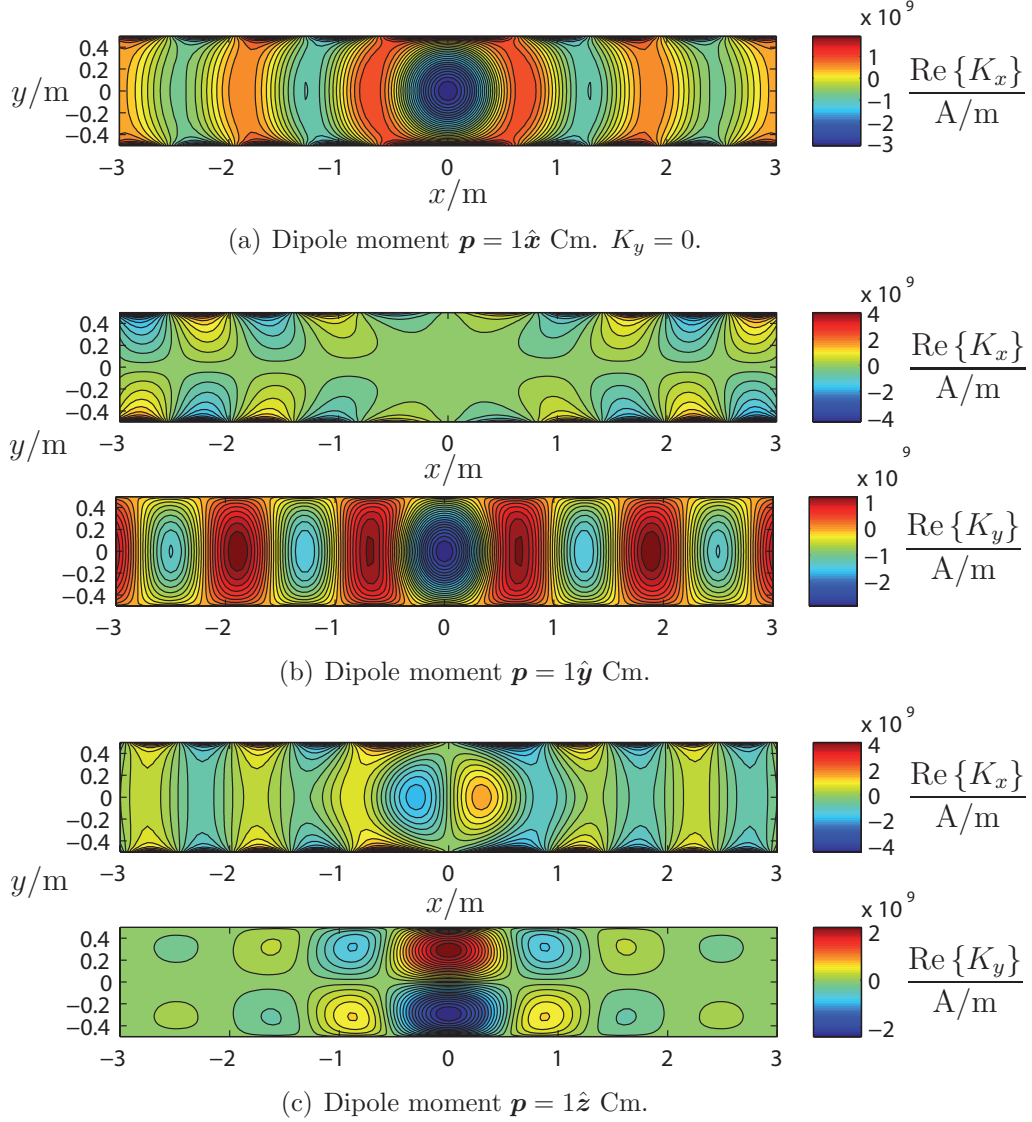
### 3.3 Simplified model for narrow strips using only the total longitudinal currents

Guided by the results in subsection 3.2, we will here make approximations that result in a considerably simplified model.

First, the transverse surface current is neglected completely, i.e. we set  $K_y = 0$  whereby  $\{d_n = 0\}_{n=0}^{\infty}$  (cf. (40)). Next, assuming that the strip is not too close to the PEC plane

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<sup>3</sup>Note that “longitudinal” here refers to the direction of the strip, the  $x$ -direction, while in Section 2 “longitudinal” refers to the decomposition direction of the fields, the  $z$ -direction.



**Figure 2:** Surface current when  $h = 0.5$  m,  $a = 1$  m. Dipole at  $\mathbf{r}_0 = 0.5\hat{z}$  m.

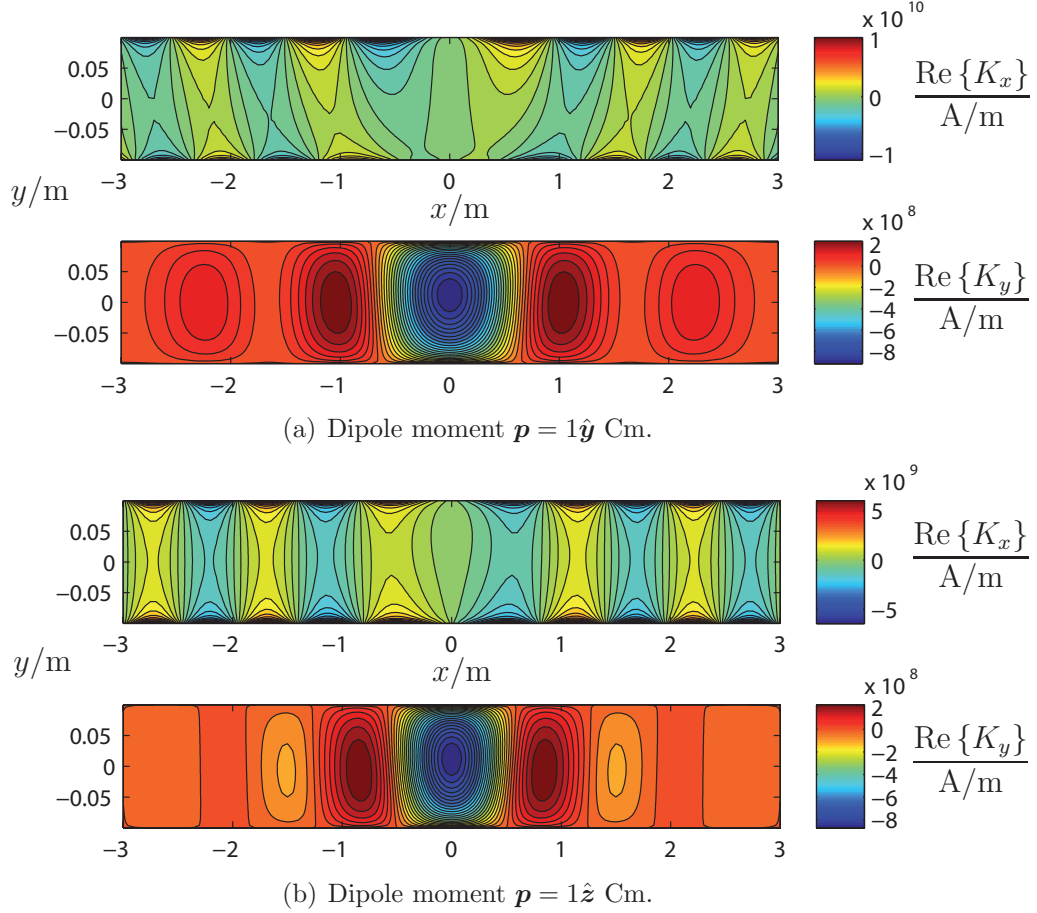
and not too close to the dipole, we neglect proximity effects and assume that in (39) the longitudinal surface current density can be described by the zeroth order coefficient only, i.e.  $\{c_n = 0\}_{n=1}^{\infty}$ . From (39) and (59), it thus follows that

$$K_x(x, y) = \frac{I(x)}{\pi h} \frac{H(h - |y|)}{\sqrt{1 - y^2/h^2}} \quad (60)$$

and, introducing the spectral current  $I(k_x) = \pi h c_0(k_x)$ , that (41) reduces to

$$K_x(\mathbf{k}_t) = I(k_x) J_0(k_y h) \quad (61)$$

In order to avoid an over-determined system, the final approximation is that we omit using (46) to enforce  $E_y = 0$  on the strip. Hence, we only use  $m = 0$  in (45) to enforce that the longitudinal electric field component  $E_x = 0$  on the strip, whereby (50) reduces



**Figure 3:** Surface current when  $h = 0.1$  m,  $a = 6$  m. Dipole at  $\mathbf{r}_0 = 0.5\hat{\mathbf{y}} + 5.5\hat{\mathbf{z}}$  m. Note that, for graphical clarity, different scalings are used on the  $x$ - and  $y$ -axes.

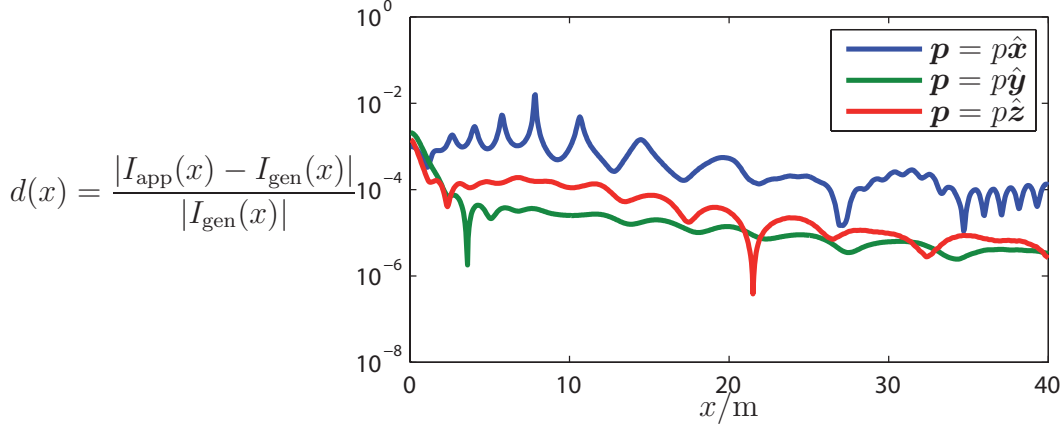
to

$$\int_{-\infty}^{\infty} E_x(k_x, k_y) J_0(k_y h) dk_y = 0 \quad (62)$$

Now, under these approximations, the general equations (55)-(57) fall out, leaving us with (54), which, as a scalar equation for the spectral current, becomes

$$I(k_x) \frac{k^2 - k_x^2}{k} \int_0^\infty \frac{1 - e^{-j2k_z a}}{k_z} J_0^2(k_y h) dk_y = \frac{j p}{\varepsilon \eta} e^{jk_x x_0} \int_0^\infty J_0(k_y h) dk_y \left\{ \begin{array}{ll} -\frac{k^2 - k_x^2}{k_z} [e^{-jk_z |a-z_0|} - e^{-jk_z (a+z_0)}] \cos(k_y y_0) & : \mathbf{p} = p\hat{\mathbf{x}} \\ \frac{k_x k_y}{k_z} [e^{-jk_z |a-z_0|} - e^{-jk_z (a+z_0)}] j \sin(k_y y_0) & : \mathbf{p} = p\hat{\mathbf{y}} \\ k_x [\text{sgn}(a - z_0) e^{-jk_z |a-z_0|} + e^{-jk_z (a+z_0)}] \cos(k_y y_0) & : \mathbf{p} = p\hat{\mathbf{z}} \end{array} \right. \quad (63)$$

To verify the approximation of using only the total longitudinal current, we define the relative difference  $d(x) = |I_{\text{app}}(x) - I_{\text{gen}}(x)| / |I_{\text{gen}}(x)|$ , where  $I_{\text{app}}(x)$  is the current obtained



**Figure 4:** Relative difference between currents obtained using the approximate method and the general method.

from using the approximate equation (63) and  $I_{\text{gen}}(x)$  is the current obtained from using the complete system of equations. In the comparison example, the geometrical parameters are the same as in Figure 3. With the dipole at  $x = 0$ , symmetry yields that we only need to consider  $x \geq 0$ , and here we consider the range  $0 \leq x \leq 40$  m. The results are shown in Figure 4, where we see that  $d(x) \ll 1$ , even in this case with a strip as wide as  $1/5$  of the wavelength.

### 3.4 Results for narrow strips compared with results for thin circular wires

In this section, we compare results for the narrow strip with results for the equivalent circular wire. With negligible proximity effects, the per length inductances and capacitances for a wire with radius  $s$  become the same as for the strip with halfwidth  $h$  if  $s = h/2$ ; see e.g. [21].

#### 3.4.1 Identification of TEM-mode in the $x$ -direction

For a wire/strip at small height (in terms of wavelengths) above the ground plane, the TEM-mode (if excited) is expected to be the dominating mode, in the field and current distributions. Note that the propagation along the wire/strip is in the  $x$ -direction, wherefore TEM here refers to field components in the  $y$ - and  $z$ -directions only.

In Appendix D, we derive the expressions (120)-(122) for the ideal TEM-mode currents on the wire excited by dipole sources in the principal directions.

In Figures 5-6, we present results for the excited currents with the strip at the heights  $a = 1.0$  m and  $a = 2.0$  m, respectively (the wavelength is 1 m). In the same figures, we also present the TEM-mode current on the equivalent circular wire, as well as the difference between the total current on the strip and the TEM-mode current on the wire.

(120) yields that  $x$ -directed dipoles will not excite TEM-modes on the wire. Correspondingly, we obtain in the expression (63) for the spectral current on the strip that for an



$x$ -directed dipole there is no pole at  $k_x = k$  (the TEM-mode wavenumber), whereby the TEM-mode is not excited. Hence, the results in Figures 5(a) and 6(a) are for higher order modes on the strip only. With  $a = 1.0$  m we see in Figures 5(b) and 5(c) that the excited strip current is explained reasonably by the wire TEM-mode current, apart from in the vicinity of the dipole. Roughly, the difference between the total current on the strip and the wire TEM-mode current is the contribution from the higher order modes on the strip. Increasing further to  $a = 2.0$  m we see in Figures 6(b) and 6(c) that, within the distances considered, the strip current is no longer dominated by the TEM-mode.

### 3.4.2 Comparison with results obtained using MoM-solver

Since the spectral method is designed to handle strips of infinite length, and commercial softwares that can handle infinitely long wires are not readily available, it is difficult to set up a proper comparison example. Nevertheless, we use the method of moment code NEC [31] to compare the results for a wire of finite length with the results using the spectral method. The finite length wire is located at  $|x| \leq 40$  m and is left open-ended.

In Figure 7 we see that with an  $x$ -directed dipole the agreement is quite satisfactory. The reason is the absence of the TEM-mode. Due to radiation, the higher order modes suffer from a much faster attenuation, resulting in quite small reflections from the open-ended wire in the NEC-model. On the other hand, in Figure 8 we see that with a  $z$ -directed dipole the agreement is far from satisfactory. Here, the TEM-mode is present, and due to its small attenuation the reflections from the open end-points of the wire creates standing waves in the NEC-model. Along the infinitely long strip there are no standing waves (but of course interference between the strip and the PEC plane).

## 4 Discussion and conclusions

In this paper, we have investigated high frequency wave propagation for the generic case of a metallic PEC strip above a PEC ground plane. Starting from a general formulation for a wide strip, we have verified the approximation of using only longitudinal currents on a narrow strip. Furthermore, we have verified the equivalence between narrow strips and thin wires, when proximity effects can be neglected.

Although generic, the case studied includes the key-parts regarding the mathematical analysis and the approximations made. Hence, it is in principle straight-forward to generalize to multiple parallel strips, multiple layers (including lossy media) and multiple dipole or distributed sources. However, when considering wide strips, requiring the complete system (54)-(57) of equations, a generalization can be rather cumbersome. On the other hand, the simplified model for narrow strips (equivalent with thin wires), based on equation (63), is more tractable under a generalization to multiple wires and/or layers. Hence, the method outlined in this paper has potential usefulness for estimating long range propagation of high frequency waves in wire structures. Such structures can be power lines and railway feeding systems, containing both over-head wires and wires submerged into ground.

## A Derivation of transversal field expressions

Here, we derive the expressions (9) and (10) for the transversal fields.

With

$$\mathbf{E}^\pm(\mathbf{k}_t, z), \mathbf{H}^\pm(\mathbf{k}_t, z) \propto e^{\mp j k_z z} \quad (64)$$

and (4) to express the fields in the source-free ( $\mathbf{J}=\mathbf{0}$ ) Maxwell's equations (1) and (2), the spectral fields fulfil

$$(\mathbf{k}_t \pm k_z \hat{\mathbf{z}}) \times \mathbf{E}^\pm = k \eta \mathbf{H}^\pm \quad (65)$$

$$(\mathbf{k}_t \pm k_z \hat{\mathbf{z}}) \times \eta \mathbf{H}^\pm = -k \mathbf{E}^\pm \quad (66)$$

Decomposing (65), (66) and the fields into longitudinal and transversal parts, we obtain

$$\mathbf{k}_t \times \mathbf{E}_t^\pm = k \eta H_z^\pm \hat{\mathbf{z}} \quad (67)$$

$$\mathbf{k}_t \times \hat{\mathbf{z}} E_z^\pm \pm k_z \hat{\mathbf{z}} \times \mathbf{E}_t^\pm = k \eta \mathbf{H}_t^\pm \quad (68)$$

$$\mathbf{k}_t \times \eta \mathbf{H}_t^\pm = -k E_z^\pm \hat{\mathbf{z}} \quad (69)$$

$$\mathbf{k}_t \times \hat{\mathbf{z}} \eta H_z^\pm \pm k_z \hat{\mathbf{z}} \times \eta \mathbf{H}_t^\pm = -k \mathbf{E}_t^\pm \quad (70)$$

Performing the operations  $\hat{\mathbf{z}} \times (68)$  and  $\hat{\mathbf{z}} \times (70)$ , we obtain

$$\mathbf{k}_t E_z^\pm \mp k_z \mathbf{E}_t = k \hat{\mathbf{z}} \times \eta \mathbf{H}_t \quad (71)$$

$$\mathbf{k}_t \eta H_z^\pm \mp k_z \eta \mathbf{H}_t = -k \hat{\mathbf{z}} \times \mathbf{E}_t \quad (72)$$

Eliminating  $\hat{\mathbf{z}} \times \mathbf{E}_t^\pm$  from (68) and (72), and using (8), we obtain (9):

$$\mathbf{E}_t^\pm = \frac{-1}{k_t^2} [\pm k_z \mathbf{k}_t E_z^\pm - k \hat{\mathbf{z}} \times \mathbf{k}_t \eta H_z^\pm], \quad (9)'$$

Similarly, eliminating  $\hat{\mathbf{z}} \times \eta \mathbf{H}_t^\pm$  from (70) and (71), and using (8), we obtain (10):

$$\eta \mathbf{H}_t^\pm = \frac{-1}{k_t^2} [\pm k_z \mathbf{k}_t \eta H_z^\pm + k \hat{\mathbf{z}} \times \mathbf{k}_t E_z^\pm], \quad (10)'$$

Note that the outlined derivation is the common way of expressing the transversal fields in e.g. a waveguiding structure, where one instead has  $\mathbf{k}_t \rightarrow j \nabla_t$  (see e.g. [26], Section 5.1).

## B The spectral fields from the electric dipole

In this appendix, we derive the expressions (23)-(25) for the functions  $A_1(\mathbf{k}_t, z)$  and  $B_1(\mathbf{k}_t, z)$ , in the spectral fields from the electric dipole.

### B.1 Spectral fields from a localised source

First, we consider the more general case with a distributed but localized source  $\mathbf{J}(\mathbf{r})$  located in the slab region between the planar surfaces  $\mathcal{S}_1$ , at  $z = z_1$ , and  $\mathcal{S}_2$ , at  $z = z_2$  ( $z_1 < z_2$ ); see Figure 9. From the losses in the medium and conservation of energy, it follows that the modes attenuate away from the slab region containing the source.

Hence, from (15)-(20) it holds for  $z \geq z_2$  that

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi^2} \int_{\mathcal{K}} [A^+(\mathbf{k}_t) \mathbf{f}^+(\mathbf{k}_t) + B^+(\mathbf{k}_t) \mathbf{g}(\mathbf{k}_t)] e^{-jk_z z} e^{-j\mathbf{k}_t \cdot \boldsymbol{\rho}} d^2 k_t \quad (73)$$

$$\eta \mathbf{H}(\mathbf{r}) = \frac{1}{4\pi^2} \int_{\mathcal{K}} [A^+(\mathbf{k}_t) \mathbf{g}(\mathbf{k}_t) - B^+(\mathbf{k}_t) \mathbf{f}^+(\mathbf{k}_t)] e^{-jk_z z} e^{-j\mathbf{k}_t \cdot \boldsymbol{\rho}} d^2 k_t \quad (74)$$

while for  $z \leq z_1$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi^2} \int_{\mathcal{K}} [A^-(\mathbf{k}_t) \mathbf{f}^-(\mathbf{k}_t) + B^-(\mathbf{k}_t) \mathbf{g}(\mathbf{k}_t)] e^{+jk_z z} e^{-j\mathbf{k}_t \cdot \boldsymbol{\rho}} d^2 k_t \quad (75)$$

$$\eta \mathbf{H}(\mathbf{r}) = \frac{1}{4\pi^2} \int_{\mathcal{K}} [-A^-(\mathbf{k}_t) \mathbf{g}(\mathbf{k}_t) + B^-(\mathbf{k}_t) \mathbf{f}^-(\mathbf{k}_t)] e^{+jk_z z} e^{-j\mathbf{k}_t \cdot \boldsymbol{\rho}} d^2 k_t \quad (76)$$

From (3) and (4), we have the completeness relation:

$$\frac{1}{4\pi^2} \int_{\mathcal{S}} e^{\pm j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} = \delta(\mathbf{k}_t - \mathbf{k}'_t) \quad (77)$$

where  $\delta()$  denotes the Dirac delta distribution. From (77), we have the corollaries

$$[\mathbf{f}^\pm(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t)] \cdot \hat{\mathbf{z}} \frac{1}{4\pi^2} \int_{\mathcal{S}} e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} = \frac{k_z k}{k_t^2 (k'_t)^2} \mathbf{k}_t \cdot \mathbf{k}'_t \delta(\mathbf{k}_t - \mathbf{k}'_t) = \frac{k_z k}{k_t^2} \delta(\mathbf{k}_t - \mathbf{k}'_t) \quad (78)$$

$$[\mathbf{f}^\pm(\mathbf{k}_t) \times \mathbf{f}^\pm(\mathbf{k}'_t)] \cdot \hat{\mathbf{z}} \frac{1}{4\pi^2} \int_{\mathcal{S}} e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} = 0 \quad (79)$$

$$[\mathbf{f}^\pm(\mathbf{k}_t) \times \mathbf{f}^\mp(\mathbf{k}'_t)] \cdot \hat{\mathbf{z}} \frac{1}{4\pi^2} \int_{\mathcal{S}} e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} = 0 \quad (80)$$

$$[\mathbf{g}(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t)] \cdot \hat{\mathbf{z}} \frac{1}{4\pi^2} \int_{\mathcal{S}} e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} = 0 \quad (81)$$

The Lorentz reciprocity theorem, see e.g. [32], yields

$$\int_{\mathcal{S}_2} [\mathbf{E} \times \mathbf{H}^{\text{test}} - \mathbf{E}^{\text{test}} \times \mathbf{H}] \cdot \hat{\mathbf{z}} d\mathbf{s} - \int_{\mathcal{S}_1} [\mathbf{E} \times \mathbf{H}^{\text{test}} - \mathbf{E}^{\text{test}} \times \mathbf{H}] \cdot \hat{\mathbf{z}} d\mathbf{s} = \int \mathbf{J} \cdot \mathbf{E}^{\text{test}} d\mathbf{v} \quad (82)$$

where  $\{\mathbf{E}^{\text{test}}, \mathbf{H}^{\text{test}}\}$  is a solution to the Maxwell's equations that is source-free in the slab region between  $\mathcal{S}_1$  and  $\mathcal{S}_2$ . It has been assumed that the contribution to the total surface integral from the closing cylindrical boundary at infinity vanishes, for which a sufficient condition is that the source is localized and that the medium is lossy.

Next, the mode coefficients will be calculated by inserting (73)-(76) into (82) and using the results (78)-(81).

With  $\mathbf{E}^{\text{test}} = \mathbf{f}^+(\mathbf{k}_t) e^{-jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}}$ ,  $\eta \mathbf{H}^{\text{test}} = \mathbf{g}(\mathbf{k}_t) e^{-jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}}$ , we obtain

$$\begin{aligned}
& \int_{\mathcal{K}} d^2 k'_t \left[ A^+(\mathbf{k}'_t) \mathbf{f}^+(\mathbf{k}'_t) \times \mathbf{g}(\mathbf{k}_t) + B^+(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}'_t) \times \mathbf{g}(\mathbf{k}_t) \right. \\
& \quad \left. - A^+(\mathbf{k}'_t) \mathbf{f}^+(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t) + B^+(\mathbf{k}'_t) \mathbf{f}^+(\mathbf{k}_t) \times \mathbf{f}^+(\mathbf{k}'_t) \right] \cdot \hat{\mathbf{z}} e^{-j2k_z z_2} \frac{1}{4\pi^2} \int_S e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} \\
& - \int_{\mathcal{K}} d^2 k'_t \left[ A^-(\mathbf{k}'_t) \mathbf{f}^-(\mathbf{k}'_t) \times \mathbf{g}(\mathbf{k}_t) + B^-(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}'_t) \times \mathbf{g}(\mathbf{k}_t) \right. \\
& \quad \left. + A^-(\mathbf{k}'_t) \mathbf{f}^+(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t) - B^-(\mathbf{k}'_t) \mathbf{f}^+(\mathbf{k}_t) \times \mathbf{f}^-(\mathbf{k}'_t) \right] \cdot \hat{\mathbf{z}} \frac{1}{4\pi^2} \int_S e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} \\
& = -\frac{2k_z k}{k_t^2} A^-(\mathbf{k}_t) = \eta \int \mathbf{J}(\mathbf{r}) \cdot \mathbf{f}^+(\mathbf{k}_t) e^{-jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}} d\mathbf{v} \tag{83}
\end{aligned}$$

With  $\mathbf{E}^{\text{test}} = \mathbf{f}^-(\mathbf{k}_t) e^{+jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}}$ ,  $\eta \mathbf{H}^{\text{test}} = -\mathbf{g}(\mathbf{k}_t) e^{+jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}}$ , we obtain

$$\begin{aligned}
& \int_{\mathcal{K}} d^2 k'_t \left[ -A^+(\mathbf{k}'_t) \mathbf{f}^+(\mathbf{k}'_t) \times \mathbf{g}(\mathbf{k}_t) - B^+(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}'_t) \times \mathbf{g}(\mathbf{k}_t) \right. \\
& \quad \left. - A^+(\mathbf{k}'_t) \mathbf{f}^-(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t) + B^+(\mathbf{k}'_t) \mathbf{f}^-(\mathbf{k}_t) \times \mathbf{f}^+(\mathbf{k}'_t) \right] \cdot \hat{\mathbf{z}} \frac{1}{4\pi^2} \int_S e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} \\
& - \int_{\mathcal{K}} d^2 k'_t \left[ -A^-(\mathbf{k}'_t) \mathbf{f}^-(\mathbf{k}'_t) \times \mathbf{g}(\mathbf{k}_t) - B^-(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}'_t) \times \mathbf{g}(\mathbf{k}_t) \right. \\
& \quad \left. + A^-(\mathbf{k}'_t) \mathbf{f}^-(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t) - B^-(\mathbf{k}'_t) \mathbf{f}^-(\mathbf{k}_t) \times \mathbf{f}^-(\mathbf{k}'_t) \right] \cdot \hat{\mathbf{z}} e^{+j2k_z z_1} \frac{1}{4\pi^2} \int_S e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} \\
& = -\frac{2k_z k}{k_t^2} A^+(\mathbf{k}_t) = \eta \int \mathbf{J}(\mathbf{r}) \cdot \mathbf{f}^-(\mathbf{k}_t) e^{+jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}} d\mathbf{v} \tag{84}
\end{aligned}$$

With  $\mathbf{E}^{\text{test}} = \mathbf{g}(\mathbf{k}_t) e^{-jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}}$ ,  $\eta \mathbf{H}^{\text{test}} = -\mathbf{f}^+(\mathbf{k}_t) e^{-jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}}$ , we obtain

$$\begin{aligned}
& \int_{\mathcal{K}} d^2 k'_t \left[ -A^+(\mathbf{k}'_t) \mathbf{f}^+(\mathbf{k}'_t) \times \mathbf{f}^+(\mathbf{k}_t) - B^+(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}'_t) \times \mathbf{f}^+(\mathbf{k}_t) \right. \\
& \quad \left. - A^+(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t) + B^+(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}_t) \times \mathbf{f}^+(\mathbf{k}'_t) \right] \cdot \hat{\mathbf{z}} e^{-j2k_z z_2} \frac{1}{4\pi^2} \int_S e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} \\
& - \int_{\mathcal{K}} d^2 k'_t \left[ -A^-(\mathbf{k}'_t) \mathbf{f}^-(\mathbf{k}'_t) \times \mathbf{f}^+(\mathbf{k}_t) - B^-(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}'_t) \times \mathbf{f}^+(\mathbf{k}_t) \right. \\
& \quad \left. + A^-(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t) - B^-(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}_t) \times \mathbf{f}^-(\mathbf{k}'_t) \right] \cdot \hat{\mathbf{z}} \frac{1}{4\pi^2} \int_S e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} \\
& = -\frac{2k_z k}{k_t^2} B^-(\mathbf{k}_t) = \eta \int \mathbf{J}(\mathbf{r}) \cdot \mathbf{g}(\mathbf{k}_t) e^{-jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}} d\mathbf{v} \tag{85}
\end{aligned}$$

With  $\mathbf{E}^{\text{test}} = \mathbf{g}(\mathbf{k}_t) e^{+jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}}$ ,  $\eta \mathbf{H}^{\text{test}} = \mathbf{f}^-(\mathbf{k}_t) e^{+jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}}$ , we obtain

$$\begin{aligned}
& \int_{\mathcal{K}} d^2 k'_t \left[ A^+(\mathbf{k}'_t) \mathbf{f}^+(\mathbf{k}'_t) \times \mathbf{f}^-(\mathbf{k}_t) + B^+(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}'_t) \times \mathbf{f}^-(\mathbf{k}_t) \right. \\
& \quad \left. - A^+(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t) + B^+(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}_t) \times \mathbf{f}^+(\mathbf{k}'_t) \right] \cdot \hat{\mathbf{z}} \frac{1}{4\pi^2} \int_{\mathcal{S}} e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} \\
& - \int_{\mathcal{K}} d^2 k'_t \left[ A^-(\mathbf{k}'_t) \mathbf{f}^-(\mathbf{k}'_t) \times \mathbf{f}^-(\mathbf{k}_t) + B^-(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}'_t) \times \mathbf{f}^-(\mathbf{k}_t) \right. \\
& \quad \left. + A^-(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}_t) \times \mathbf{g}(\mathbf{k}'_t) - B^-(\mathbf{k}'_t) \mathbf{g}(\mathbf{k}_t) \times \mathbf{f}^-(\mathbf{k}'_t) \right] \cdot \hat{\mathbf{z}} e^{+j2k_z z_1} \frac{1}{4\pi^2} \int_{\mathcal{S}} e^{j(\mathbf{k}_t - \mathbf{k}'_t) \cdot \boldsymbol{\rho}} d\mathbf{s} \\
& = -\frac{2k_z k}{k_t^2} B^+(\mathbf{k}_t) = \eta \int \mathbf{J}(\mathbf{r}) \cdot \mathbf{g}(\mathbf{k}_t) e^{+jk_z z} e^{j\mathbf{k}_t \cdot \boldsymbol{\rho}} d\mathbf{v}
\end{aligned} \tag{86}$$

Summarizing, and using (12) and (13), we obtain the relations

$$A^\pm(\mathbf{k}_t) = -\frac{\eta}{2k_z k} (k_z \mathbf{k}_t \pm k_t^2 \hat{\mathbf{z}}) \cdot \int \mathbf{J}(\mathbf{r}) e^{j(\mathbf{k}_t \cdot \boldsymbol{\rho} \pm k_z z)} d\mathbf{v} \tag{87}$$

$$B^\pm(\mathbf{k}_t) = -\frac{\eta}{2k_z} (\hat{\mathbf{z}} \times \mathbf{k}_t) \cdot \int \mathbf{J}(\mathbf{r}) e^{j(\mathbf{k}_t \cdot \boldsymbol{\rho} \pm k_z z)} d\mathbf{v} \tag{88}$$

An electric dipole  $\mathbf{p}$  at the location  $\mathbf{r}_0 = \boldsymbol{\rho}_0 + z_0 \hat{\mathbf{z}}$  gives rise to the current density

$$\mathbf{J}(\mathbf{r}) = j\omega \mathbf{p} \delta(\mathbf{r} - \mathbf{r}_0) \tag{89}$$

where  $\delta()$  denotes the Dirac delta distribution.

Using (89) in (87) and (88), we obtain for the three principal orientations of the dipole

$$\mathbf{p} = p\hat{\mathbf{x}} \Rightarrow \begin{cases} A^\pm(\mathbf{k}_t) = -\frac{jpk_x}{2\varepsilon} e^{j(\mathbf{k}_t \cdot \boldsymbol{\rho}_0 \pm k_z z_0)} \\ B^\pm(\mathbf{k}_t) = \frac{jpk_y}{2\varepsilon k_z} e^{j(\mathbf{k}_t \cdot \boldsymbol{\rho}_0 \pm k_z z_0)} \end{cases} \tag{90}$$

$$\mathbf{p} = p\hat{\mathbf{y}} \Rightarrow \begin{cases} A^\pm(\mathbf{k}_t) = -\frac{jpk_y}{2\varepsilon} e^{j(\mathbf{k}_t \cdot \boldsymbol{\rho}_0 \pm k_z z_0)} \\ B^\pm(\mathbf{k}_t) = -\frac{jpk_x}{2\varepsilon k_z} e^{j(\mathbf{k}_t \cdot \boldsymbol{\rho}_0 \pm k_z z_0)} \end{cases} \tag{91}$$

$$\mathbf{p} = p\hat{\mathbf{z}} \Rightarrow \begin{cases} A^\pm(\mathbf{k}_t) = \pm \frac{jpk_t^2}{2\varepsilon k_z} e^{j(\mathbf{k}_t \cdot \boldsymbol{\rho}_0 \pm k_z z_0)} \\ B^\pm(\mathbf{k}_t) = 0 \end{cases} \tag{92}$$

Finally, with

$$A_1(\mathbf{k}_t, z) = A^\pm(\mathbf{k}_t) e^{\mp jk_z z}, \quad B_1(\mathbf{k}_t, z) = B^\pm(\mathbf{k}_t) e^{\mp jk_z z} \tag{93}$$

we obtain the expressions (23)-(25).

## C Numerical evaluation of spectral integrals

Here, we describe the numerical evaluation of the spectral integrals in (54)-(57).

Since the integrals run over the semi-infinite interval  $0 < k_y < \infty$  and are not open for easy analytic evaluation, they must be approximated over a finite interval  $0 < k_y < k_{y,\max}$ . To increase the rate of convergence with increasing  $k_{y,\max}$ , we identify and subtract off leading behaviours of the integrands for large  $k_y$  that can be integrated analytically.

First, the longitudinal wavenumber is

$$k_z = \sqrt{k^2 - k_x^2 - k_y^2} \quad (94)$$

With  $\text{Im}\{k_z\} < 0$  and  $0 < k_y < \infty$ , it holds for  $k_y \gg |k_x|, |k|$  that  $k_z \approx -jk_y$ .

Hence, for large  $k_y$  we have in the left-hand sides of (54)-(57) integrands that behave as

$$\frac{1 - e^{-j2k_z a}}{k_z} J_{\nu_1}(k_y h) J_{\nu_2}(k_y h) \approx \frac{1 - e^{-2k_y a}}{-jk_y} J_{\nu_1}(k_y h) J_{\nu_2}(k_y h) \approx \frac{j}{k_y} J_{\nu_1}(k_y h) J_{\nu_2}(k_y h) \quad (95)$$

$$\frac{(1 - e^{-j2k_z a})(k^2 - k_y^2)}{k_y^2 k_z} J_{\nu_1}(k_y h) J_{\nu_2}(k_y h) \approx \frac{-j}{k_y} J_{\nu_1}(k_y h) J_{\nu_2}(k_y h) \quad (96)$$

From [24], formula 6.574, we have the following results for non-negative integers  $m$  and  $n$ :

$$\int_0^\infty \frac{J_{2m}(k_y h) J_{2n}(k_y h)}{k_y} dk_y = \frac{\delta_{mn}}{2(m+n)}, \quad (m, n) \neq (0, 0) \quad (97)$$

$$\int_0^\infty \frac{J_{2m+1}(k_y h) J_{2n+1}(k_y h)}{k_y} dk_y = \frac{\delta_{mn}}{2(m+n+1)} \quad (98)$$

Using (97), the integrals in (54) and (55) are approximated as

$$\begin{aligned} & \int_0^\infty \frac{1 - e^{-j2k_z a}}{k_z} J_{2m}(k_y h) J_{2n}(k_y h) dk_y \\ & \approx \int_0^{k_{y,\max}} \left[ \frac{1 - e^{-j2k_z a}}{k_z} - \frac{j}{k_y} \right] J_{2m}(k_y h) J_{2n}(k_y h) dk_y + \frac{j \delta_{mn}}{2(m+n)}, \quad (m, n) \neq (0, 0) \end{aligned} \quad (99)$$

$$\begin{aligned} & \int_0^\infty \frac{(1 - e^{-j2k_z a})(k^2 - k_y^2)}{k_y^2 k_z} J_{2m}(k_y h) J_{2n}(k_y h) dk_y \\ & \approx \int_0^{k_{y,\max}} \left[ \frac{(1 - e^{-j2k_z a})(k^2 - k_y^2)}{k_y^2 k_z} + \frac{j}{k_y} \right] J_{2m}(k_y h) J_{2n}(k_y h) dk_y - \frac{j \delta_{mn}}{2(m+n)} \end{aligned} \quad (100)$$

For the case  $m = n = 0$ , the first integral in (54) is approximated as

$$\begin{aligned} & \int_0^\infty \frac{1 - e^{-j2k_z a}}{k_z} J_0^2(k_y h) dk_y \approx \int_0^{1/h} \frac{1 - e^{-j2k_z a}}{k_z} J_0^2(k_y h) dk_y \\ & + \int_{1/h}^{k_{y,\max}} \left[ \frac{1 - e^{-j2k_z a}}{k_z} - \frac{j}{k_y} \right] J_0^2(k_y h) dk_y + \int_1^\infty \frac{J_0^2(u)}{u} du, \end{aligned} \quad (101)$$

where  $\int_1^\infty \frac{J_0^2(u)}{u} du \approx 0.3438831082$ , evaluated as a generalized hypergeometric function. Using (98), the integrals in (56) and (57) are approximated as

$$\begin{aligned} & \int_0^\infty \frac{1 - e^{-j2k_z a}}{k_z} J_{2m+1}(k_y h) J_{2n+1}(k_y h) dk_y \\ & \approx \int_0^{k_{y,\max}} \left[ \frac{1 - e^{-j2k_z a}}{k_z} - \frac{j}{k_y} \right] J_{2m+1}(k_y h) J_{2n+1}(k_y h) dk_y + \frac{j \delta_{mn}}{2(m+n+1)} \end{aligned} \quad (102)$$

$$\begin{aligned} & \int_0^\infty \frac{(1 - e^{-j2k_z a}) (k^2 - k_y^2)}{k_y^2 k_z} J_{2m+1}(k_y h) J_{2n+1}(k_y h) dk_y \\ & \approx \int_0^{k_{y,\max}} \left[ \frac{(1 - e^{-j2k_z a}) (k^2 - k_y^2)}{k_y^2 k_z} + \frac{j}{k_y} \right] J_{2m+1}(k_y h) J_{2n+1}(k_y h) dk_y - \frac{j \delta_{mn}}{2(m+n+1)} \end{aligned} \quad (103)$$

In the right hand sides of (54)-(56), the exponential functions typically guaranty rapid convergence of the integrals, except for small values of  $a - z_0$ , i.e. when the dipole source is close to the plane containing the strip. To account for such cases, we for  $x$ -directed dipoles extract the following explicit integral for the leading behaviours ([24] 6.621):

$$\int_0^\infty \frac{e^{-k_y b}}{k_y} J_m(k_y h) dk_y = \frac{1}{m} \left( \frac{h}{2b} \right)^m F\left( \frac{m}{2}, \frac{m+1}{2}; m+1; -\left( \frac{h}{b} \right)^2 \right), \quad m = 1, 2, \dots \quad (104)$$

where  $F()$  is the hypergeometric function. For  $y$ - and  $z$ -directed dipoles we extract the following explicit integral for the leading behaviours ([24] 6.611):

$$\int_0^\infty e^{-k_y b} J_m(k_y h) dk_y = \frac{[\sqrt{b^2 + h^2} - b]^m}{h^m \sqrt{b^2 + h^2}}, \quad m = 0, 1, \dots \quad (105)$$

In both (104) and (105), the complex variable  $b$  has  $\text{Re}\{b\} = b \mp z_0$  and  $\text{Im}\{b\} = \pm y_0$ .

The integrals in (54)-(57) have been computed numerically by the routine “quadl” in the Matlab software. We notice that the integrands have poles where  $k_z = 0$ . By considering the medium as generally lossy, the constitutive parameters have non-zero imaginary parts, moving the poles off the real axis. However, in the near lossless limit these poles are close to real axis, causing numerical problems in the integrals. To alleviate this problem, the numerical approximations of the integrals in (58) are calculated by deforming the integration path to the contour shown in Figure 10.

With  $\Delta k_x \gg -\text{Im}\{k\}$ , we obtain better numerical stability for the integrals in (54)-(57), and for the subsequent integrals in (58). The upper limit on  $\Delta k_x$  is in practice determined by the acceptable exponential growth in the term  $e^{i k_x x}$  (see (58)).

## D The TEM-mode current along an $x$ -directed circular wire

In this appendix, we derive the expressions for the TEM-mode current on a circular wire, used in Section 3.4.1 for comparison purposes.

Consider a PEC circular wire oriented in the  $x$ -direction. The wire has radius  $s$  and is centered at  $(y = 0, z = a)$  above the PEC ground plane ( $s < a$ ). Note that in the present context,  $y$  and  $z$  are the transversal directions, subscripted with  $t$ , while  $x$  is the longitudinal direction.

Applying (82) in the present context, the TEM-mode excitation coefficients become

$$D_0^\pm = -\frac{\int_{\mathcal{V}} \mathbf{J} \cdot \mathbf{E}_t e^{\pm jkx} dv}{2 \int_{\mathcal{A}} (\mathbf{E}_t \times \mathbf{H}_t) \cdot \hat{\mathbf{x}} da} \quad (106)$$

where  $\mathcal{V}$  is the volume containing the source and  $\mathcal{A}$  is the halfplane  $(z > 0) \cap (-\infty < y < \infty)$  except for the cross-section of the wire.  $\mathbf{E}_t(y, z)$  and  $\mathbf{H}_t(y, z)$  are the modal field patterns in the cross-sectional plane.

From the TEM-mode relation  $\mathbf{H}_t = \eta^{-1} \hat{\mathbf{x}} \times \mathbf{E}_t$  (where  $\hat{\mathbf{x}} \cdot \mathbf{E}_t = 0$ ), we obtain

$$D_0^\pm = -\frac{\eta}{2} \frac{\int_{\mathcal{V}} \mathbf{J} \cdot \mathbf{E}_t e^{\pm jkx} dv}{\int_{\mathcal{A}} E_t^2 da} \quad (107)$$

Introducing, the scalar potential function  $\Phi(y, z)$ ,  $\mathbf{E}_t = -\nabla_t \Phi$  and  $\nabla_t \cdot \mathbf{E}_t = \nabla_t^2 \Phi = 0$  together with the Gauss' theorem in the plane yields

$$\int_{\mathcal{A}} E_t^2 da = \int_{\mathcal{A}} |\nabla_t \Phi|^2 da = \int_{\mathcal{A}} [\nabla_t \cdot (\Phi \nabla_t \Phi) - \Phi \nabla_t^2 \Phi] da = \oint_{\mathcal{C}} \Phi \hat{\mathbf{n}} \cdot \nabla_t \Phi dl \quad (108)$$

where  $\mathcal{C} = \mathcal{C}_0 + \mathcal{C}_\infty + \mathcal{C}_s$ , in which  $\mathcal{C}_0$  is the line  $z = 0$  that is closed with the curve  $\mathcal{C}_\infty$  at infinity (with  $z > 0$ ), and  $\mathcal{C}_s$  is the circumference of the wire. With  $\Phi = 0$  on  $\mathcal{C}_0$  and  $\mathcal{C}_\infty$ , with the constant value  $\Phi = \Phi_s$  on  $\mathcal{C}_s$ , and the boundary condition  $\sigma = \varepsilon \hat{\mathbf{n}} \cdot \nabla_t \Phi$  for the surface charge density, we obtain

$$\int_{\mathcal{A}} E_t^2 da = \Phi_s \oint_{\mathcal{C}_s} \frac{\sigma}{\varepsilon} dl = \Phi_s \frac{\lambda}{\varepsilon} \quad (109)$$

where  $\lambda$  is the line charge density on the wire.

By standard image theory [34], we obtain

$$\Phi(y, z) = \frac{\lambda}{2\pi\varepsilon} \ln \sqrt{\frac{y^2 + z^2 + a^2 - s^2 + 2z\sqrt{a^2 - s^2}}{y^2 + z^2 + a^2 - s^2 - 2z\sqrt{a^2 - s^2}}} \quad (110)$$

On the wire, i.e. when  $y^2 + (z - a)^2 = s^2$ , (110) yields

$$\Phi_s = \frac{\lambda}{2\pi\varepsilon} \ln \frac{a + \sqrt{a^2 - s^2}}{s} = \frac{\lambda}{2\pi\varepsilon} \operatorname{arccosh}\left(\frac{a}{s}\right) \quad (111)$$



From (110), we obtain

$$\mathbf{E}_t(y, z) = -\nabla_t \Phi(y, z) = \frac{\lambda}{2\pi\varepsilon} \mathbf{e}(y, z) \quad (112)$$

where

$$\mathbf{e}(y, z) = \frac{y\hat{\mathbf{y}} + (z - \sqrt{a^2 - s^2})\hat{\mathbf{z}}}{y^2 + (z - \sqrt{a^2 - s^2})^2} - \frac{y\hat{\mathbf{y}} + (z + \sqrt{a^2 - s^2})\hat{\mathbf{z}}}{y^2 + (z + \sqrt{a^2 - s^2})^2} \quad (113)$$

With the dipole  $\mathbf{p}$  at  $\mathbf{r}_0$ , yielding  $\mathbf{J}(\mathbf{r}) = j\omega\mathbf{p}\delta(\mathbf{r} - \mathbf{r}_0)$ , it follows from (107), (109), (111) and (112) that

$$D_0^\pm = -\frac{\eta\varepsilon j\omega\mathbf{p} \cdot \mathbf{e}(y_0, z_0)}{2\lambda\text{arccosh}(a/s)} e^{\pm jkx_0} \quad (114)$$

Hence, the  $\mathbf{H}$ -field becomes

$$\begin{aligned} \mathbf{H} &= \pm D_0^\pm H(x - x_0) e^{\mp jkx} \mathbf{H}_t(y, z) \\ &= -\text{sgn}(x - x_0) \frac{\eta\varepsilon j\omega\mathbf{p} \cdot \mathbf{e}(y_0, z_0)}{2\lambda\text{arccosh}(a/s)} e^{\mp jk|x-x_0|} \mathbf{H}_t(y, z) \end{aligned} \quad (115)$$

To determine the current distribution along the wire, we must find the modal current associated with

$$\begin{aligned} \mathbf{H}_t(y, z) &= \frac{1}{\eta} \hat{\mathbf{x}} \times \mathbf{E}_t(y, z) \\ &= \frac{\lambda}{2\pi\varepsilon\eta} \left[ \frac{y\hat{\mathbf{z}} - (z - \sqrt{a^2 - s^2})\hat{\mathbf{y}}}{y^2 + (z - \sqrt{a^2 - s^2})^2} - \frac{y\hat{\mathbf{z}} - (z + \sqrt{a^2 - s^2})\hat{\mathbf{y}}}{y^2 + (z + \sqrt{a^2 - s^2})^2} \right] \end{aligned} \quad (116)$$

On the ground plane, the surface current density becomes

$$\mathbf{K}(y) = \hat{\mathbf{z}} \times \mathbf{H}_t(y, z = 0) = -\frac{\lambda}{\pi\varepsilon\eta} \cdot \frac{\sqrt{a^2 - s^2}}{y^2 + a^2 - s^2} \hat{\mathbf{x}} \quad (117)$$

which yields that the oppositely directed current on the wire becomes

$$I_s = -\int_{-\infty}^{\infty} \mathbf{K}(y) \cdot \hat{\mathbf{x}} dy = \frac{2\lambda}{\pi\varepsilon\eta} \int_0^{\infty} \frac{\sqrt{a^2 - s^2}}{y^2 + a^2 - s^2} dy = \frac{\lambda}{\varepsilon\eta} \quad (118)$$

Hence, the TEM-mode current distribution on the wire becomes

$$\begin{aligned} I_{\text{TEM}}(x) &= -\text{sgn}(x - x_0) \frac{\eta\varepsilon j\omega\mathbf{p} \cdot \mathbf{e}(y_0, z_0)}{2\lambda\text{arccosh}(a/s)} e^{\mp jk|x-x_0|} I_s \\ &= -\text{sgn}(x - x_0) \frac{j\omega\mathbf{p} \cdot \mathbf{e}(y_0, z_0)}{2\text{arccosh}(a/s)} e^{\mp jk|x-x_0|} \end{aligned} \quad (119)$$

Explicitly, for the three principal directions of the dipole, it follows from (113) and (119) that

$$\mathbf{p} = p\hat{\mathbf{x}} \Rightarrow I_{\text{TEM}}(x) = 0 \quad (120)$$

$$\begin{aligned} \mathbf{p} &= p\hat{\mathbf{y}} \Rightarrow \\ I_{\text{TEM}}(x) &= -\frac{j\omega p \operatorname{sgn}(x - x_0)}{2\operatorname{arccosh}(a/s)} \left[ \frac{y_0}{y_0^2 + (z_0 - \sqrt{a^2 - s^2})^2} - \frac{y_0}{y^2 + (z_0 + \sqrt{a^2 - s^2})^2} \right] \end{aligned} \quad (121)$$

$$\begin{aligned} \mathbf{p} &= p\hat{\mathbf{z}} \Rightarrow \\ I_{\text{TEM}}(x) &= -\frac{j\omega p \operatorname{sgn}(x - x_0)}{2\operatorname{arccosh}(a/s)} \left[ \frac{z_0 - \sqrt{a^2 - s^2}}{y_0^2 + (z_0 - \sqrt{a^2 - s^2})^2} - \frac{z_0 + \sqrt{a^2 - s^2}}{y_0^2 + (z_0 + \sqrt{a^2 - s^2})^2} \right] \end{aligned} \quad (122)$$

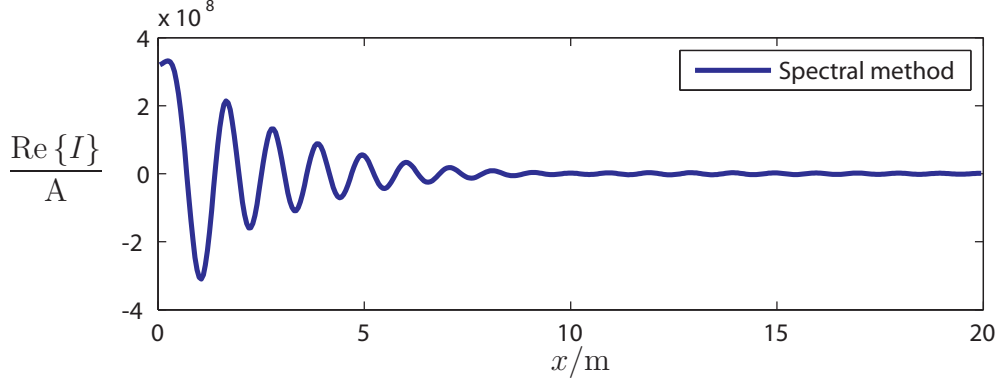
The expressions (121) and (122) hold for any wire radius  $0 < s < a$ , but if  $s \ll a$ ,  $|a - z_0|$ , we can use  $\sqrt{a^2 - s^2} \approx a$  and  $\operatorname{arccosh}(a/s) \approx \ln(2a/s)$ , cf. (111).

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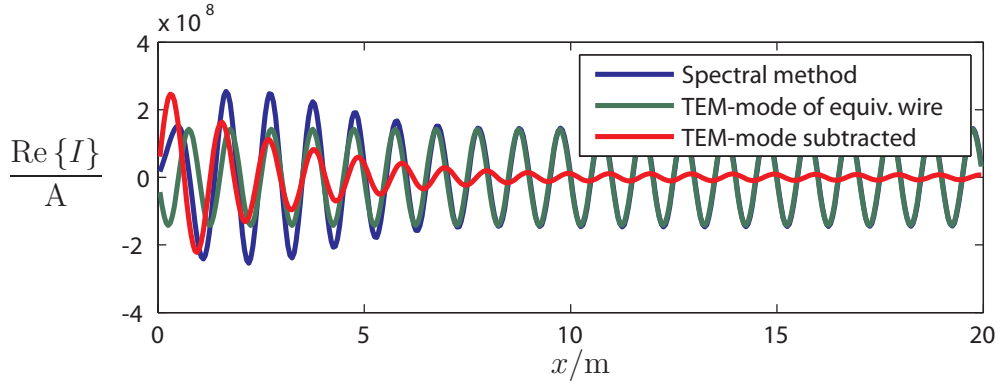
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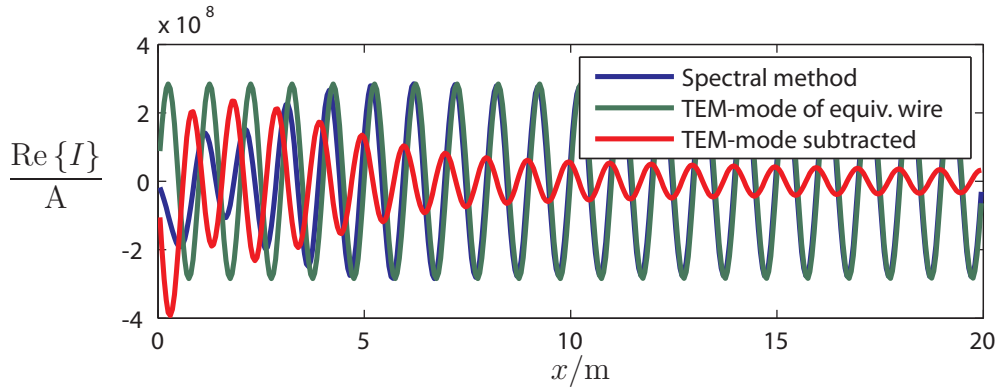
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(a) Dipole moment  $\mathbf{p} = 1\hat{\mathbf{x}}$  Cm.

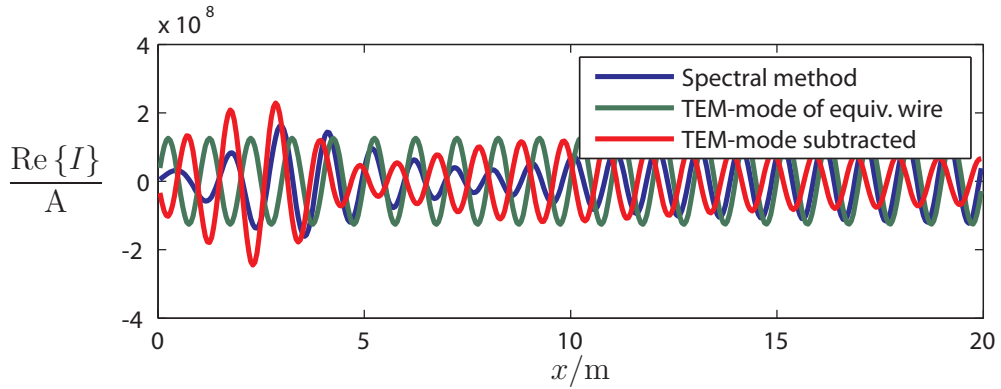
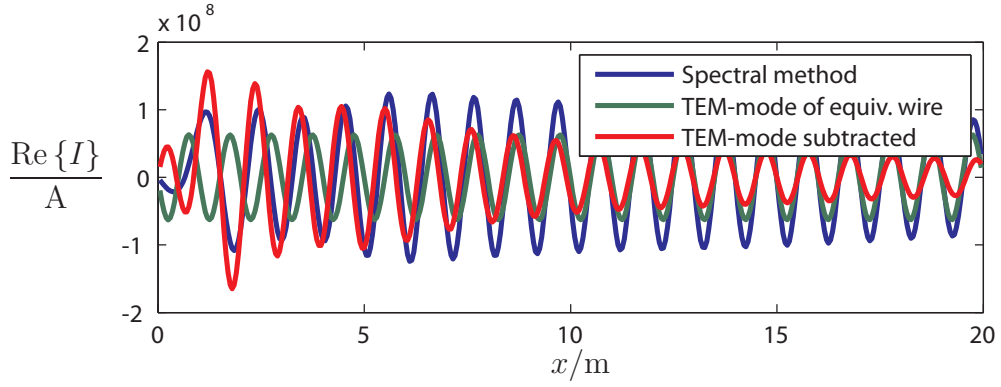
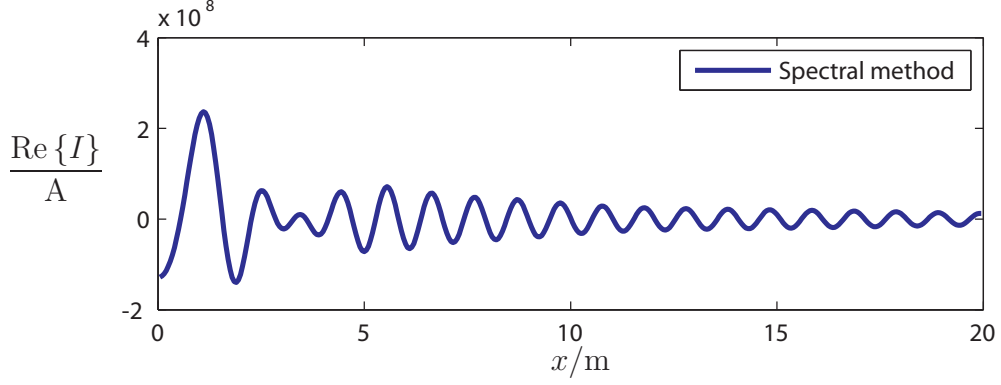


(b) Dipole moment  $\mathbf{p} = 1\hat{\mathbf{y}}$  Cm.

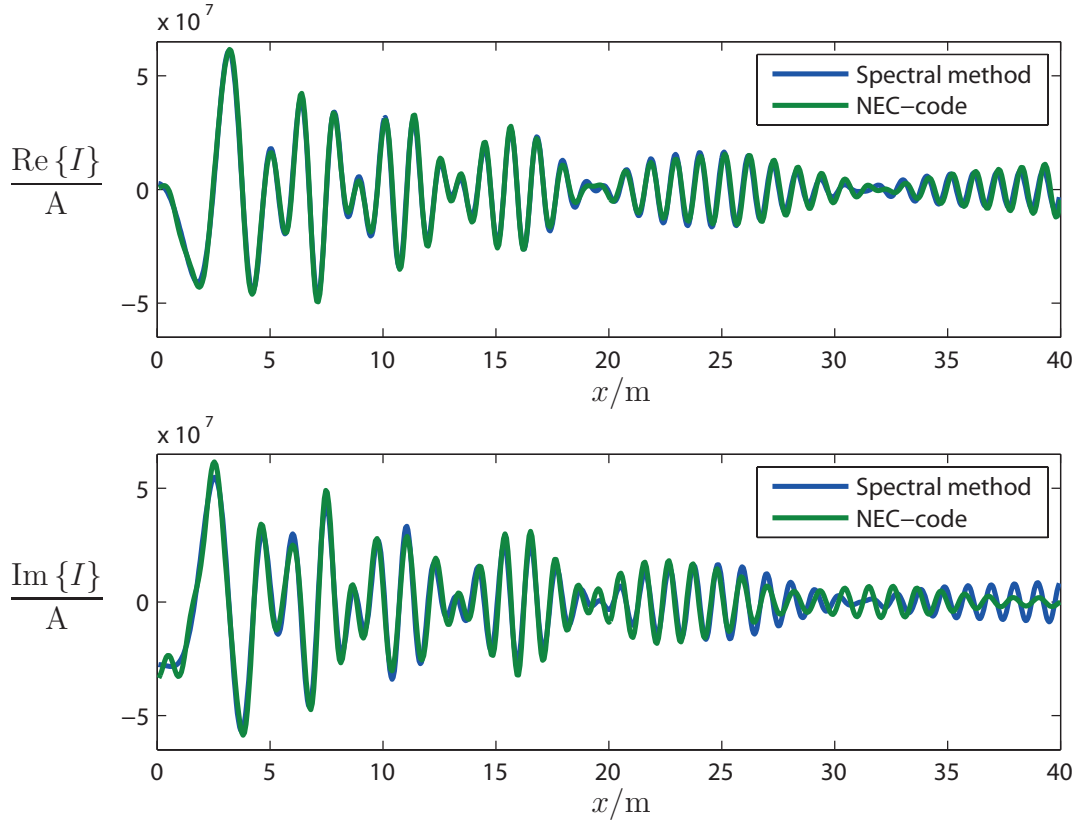


(c) Dipole moment  $\mathbf{p} = 1\hat{\mathbf{z}}$  Cm.

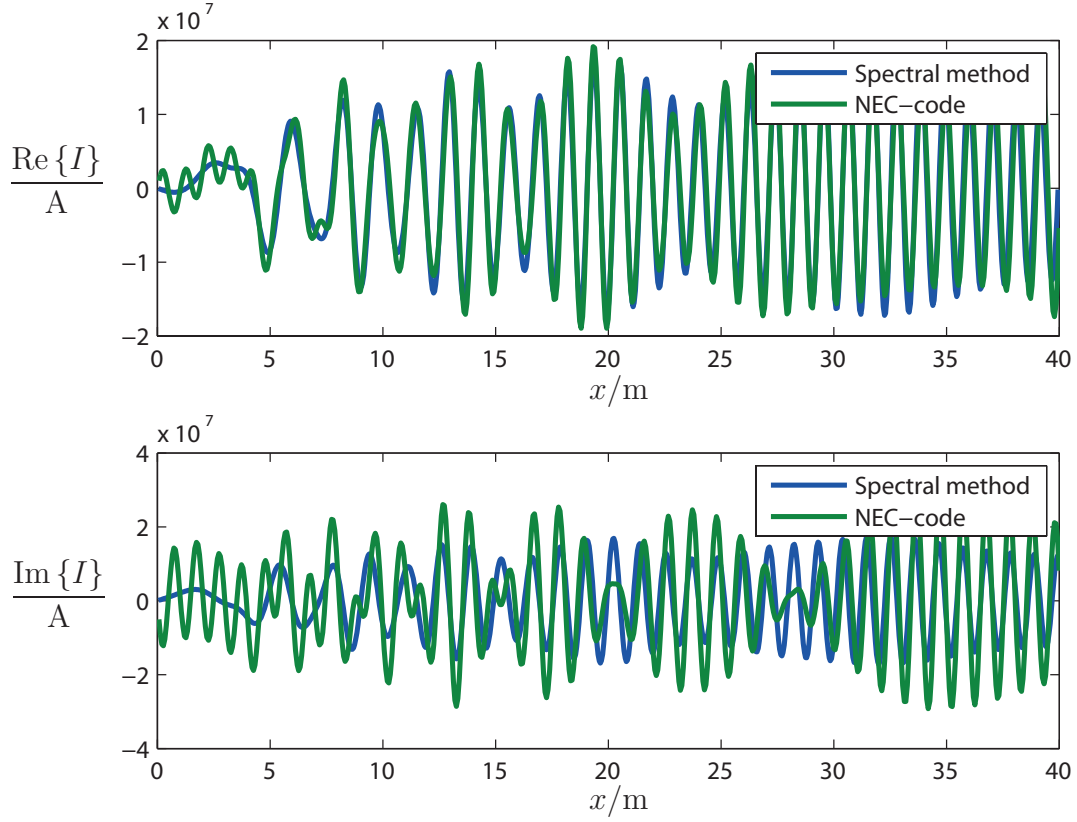
**Figure 5:** Strip current when  $h = 0.02$  m,  $a = 1$  m. Dipole at  $\mathbf{r}_0 = 0.5\hat{\mathbf{y}} + 0.5\hat{\mathbf{z}}$  m.



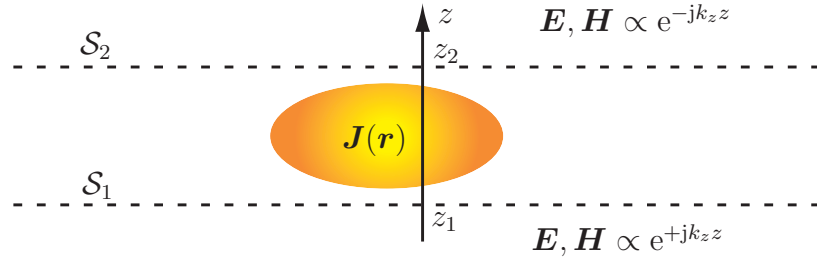
**Figure 6:** Strip current when  $h = 0.02$  m,  $a = 2$  m. Dipole at  $\mathbf{r}_0 = \hat{\mathbf{y}} + \hat{\mathbf{z}}$  m.



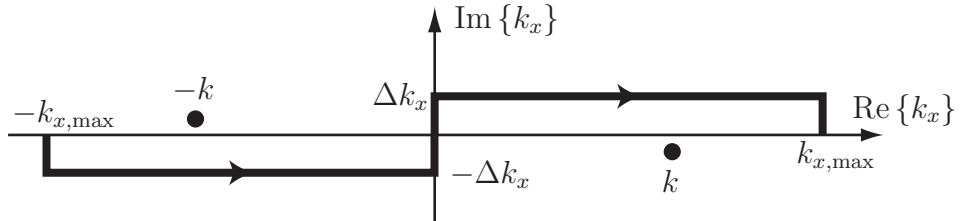
**Figure 7:** Strip current when  $h = 0.02$  m,  $a = 6$  m. Dipole at  $\mathbf{r}_0 = 6\hat{\mathbf{y}} + 5.5\hat{\mathbf{z}}$  m. Dipole moment  $\mathbf{p} = 1\hat{\mathbf{x}}$  Cm. The reference results from the NEC-code are for an open-ended wire at  $|x| \leq 40$  m.



**Figure 8:** Strip current when  $h = 0.02$  m,  $a = 6$  m. Dipole at  $\mathbf{r}_0 = 6\hat{\mathbf{y}} + 5.5\hat{\mathbf{z}}$  m. Dipole moment  $\mathbf{p} = 1\hat{\mathbf{z}}$  Cm. The reference results from the NEC-code are for an open-ended wire at  $|x| \leq 40$  m.



**Figure 9:** Geometry utilized when applying the Lorentz reciprocity theorem (82).



**Figure 10:** Integration contour in the numerical approximation of (58).